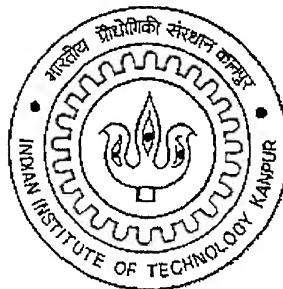


# Analysis of Strutted Deep Excavations by Finite Element Method

*A Thesis Submitted*  
in Partial Fulfillment of the Requirements  
for the Degree of  
**Master of Technology**

*by*  
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*to the*

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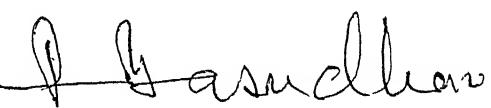
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## CERTIFICATE

It is certified that the work contained in this thesis entitled "*Analysis of Strutted Deep Excavations by Finite Element Method*", has been carried out by *Mr. Venkata Rao Bodapati*, under my supervision and that this work has not been submitted elsewhere for a degree.



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# Abstract

Deep excavations become necessary for foundations of high rise buildings, construction of tunnels, subway stations, industrial substructures and provide under ground space in the urban environments. Excavation will cause a lateral and surface settlements of the ground around the excavation zone. Controlling of the ground movements is necessary in built-up areas to protect the adjacent structures and it is a legal necessity. Without adequate lateral support the deep excavations will certainly cause the loss of bearing capacity, settlements, or lateral movements to existing structures.

The present study is focussed on the parametric studies on the lateral struttured deep excavations. Finite element package NASTRAN has been used for the analysis. The soil non homogeneity is considered in the analysis. Two dimensional plane strain analysis has been carried out and results obtained are compared with the results reported in the literature. Three dimensional analysis is carried out and the results are compared with two dimensional plane strain analysis results. Effect of various parameters like elastic modulus, strut stiffness, wall stiffness, depth to firm layer, excavation width, length to width ratio, strut spacing, and water table effect have been studied. It has been observed that for most of the parameters, the variation trend is similar and the maximum wall deflections are also close to each other in both the cases of two dimensional and three dimensional analysis. However the maximum surface settlements obtained from the three dimensional analysis values are low as compared to two dimensional plane strain results.

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# Contents

<b>List of Figures</b>	<b>iii</b>	
<b>List of Tables</b>	<b>v</b>	
<b>Nomenclature</b>	<b>vi</b>	
<b>1</b>	<b><i>Introduction</i></b>	<b>1</b>
1.1	General . . . . .	1
1.2	Literature Review . . . . .	2
1.2.1	Analytical Methods. . . . .	2
1.2.2	Observational Data from Case Studies . . . . .	6
1.2.3	Case studies using Reinforcing Techniques . . . . .	6
1.2.4	Case Studies using Effective-Stress Path Approach. . . . .	7
1.2.5	Studies on 3-D Modeling . . . . .	7
1.3	Scope of Present Investigation . . . . .	8
1.4	Organization of the Thesis. . . . .	9
<b>2</b>	<b><i>Formulation</i></b>	<b>10</b>
2.1	General . . . . .	10
2.2	Finite Element Method . . . . .	10
2.3	Formulation . . . . .	11
2.4	Finite Element Model . . . . .	14
2.4.1	Two Dimensional Model . . . . .	14
2.4.2	Three Dimensional Model . . . . .	17
2.4.3	Variation of Elastic Modulus with Depth . . . . .	20
2.4.4	Details of Package used . . . . .	21
2.4.5	Assumptions Made in the Analysis . . . . .	22
2.4.6	Method of Approach and Construction Sequence . . . . .	23
2.5	Problems Considered . . . . .	23

<b>3 Results and Discussions</b>	<b>25</b>
3.1 General . . . . .	25
3.2 Effect of Specific Parameters . . . . .	26
3.3 Relationship between Maximum Wall Movements and Maximum Surface Settlements . . . . .	26
3.4 Surface Settlement and Deflected Wall Profiles . . . . .	27
3.5 Validation of 2-D Analysis . . . . .	27
3.5.1 Effect of Soil Modulus . . . . .	29
3.5.2 Effect of Strut Stiffness . . . . .	30
3.5.3 Effect of Wall Stiffness . . . . .	31
3.5.4 Effect of Depth of Firm Layer . . . . .	31
3.5.5 Effect of Excavation Width . . . . .	32
3.5.6 Effect of Surcharge Load . . . . .	32
3.5.7 Effect of Strut Spacing . . . . .	32
3.5.8 Effect of Water Table . . . . .	32
3.6 3-D Analysis and Validation of 2-D plane strain analysis . . . . .	40
3.6.1 Effect of Soil Modulus . . . . .	41
3.6.2 Effect of Strut Stiffness . . . . .	41
3.6.3 Effect of Wall Stiffness . . . . .	41
3.6.4 Effect of Surcharge Load on Ground Surface . . . . .	42
3.6.5 Effect of Spacing of Struts . . . . .	42
3.6.6 Effect of Length to Width Ratio of Excavation . . . . .	42
<b>4 Conclusions and Scope for Future Study</b>	<b>49</b>
4.1 General . . . . .	49
4.2 Conclusions . . . . .	49
4.3 Scope for Future Study . . . . .	50
<b>Bibliography</b>	<b>51</b>

# List of Figures

2.1	Two Dimensional Model with Descretization . . . . .	15
2.2	Eight Noded Quadrilateral Element and Two Noded Spring Element . . . . .	17
2.3	3-D model with descritisation . . . . .	18
2.4	Twenty Noded Brick Element with Coordinate System . . . . .	19
2.5	Elastic Modulus of Soil Linearly Varying with Depth . . . . .	21
2.6	3-D Geometry of excavation . . . . .	24
3.1	Wall Movement Profiles with Various Modulus Multiplier Values (M) . . . . .	28
3.2	Surface Settlement Profiles for Various Modulus Multiplier Values (M) . . . . .	29
3.3	Effect of Soil Modulus Multiplier(M) on Maximum Wall Movement . . . . .	33
3.4	Effect of Soil Modulus Multiplier(M)on Maximum Surface Settlement . . . . .	34
3.5	Effect of Strut Stiffness on Maximum Wall Movement . . . . .	34
3.6	Effect of Strut Stiffness on Maximum Surface Settlement . . . . .	35
3.7	Effect of Wall Stiffness on Maximum Wall Movement . . . . .	35
3.8	Effect of Wall Stiffness on Maximum Surface Settlement . . . . .	36
3.9	Effect of Depth to Underlying Firm Layer on Maximum Wall Movement . . . . .	36
3.10	Effect of Depth to Underlying Firm Layer on Maximum Surface Settlement . . . . .	37
3.11	Effect of Excavation Width on Maximum Wall Movement . . . . .	37
3.12	Effect of Excavation Width on Maximum Surface Settlement . . . . .	38
3.13	Effect of Surcharge Loads on Maximum Wall Movements and Surface Settlements . . . . .	38
3.14	Effect of Spacing of Struts on Maximum Wall Movement and Surface Settlement . . . . .	39
3.15	Effect of Water Table on Maximum Wall Movement and Surface Settlement . . . . .	39
3.16	Effect of Soil Modulus Multiplier(M) on Maximum Wall Movement and Surface Settlement . . . . .	43
3.17	Effect of Soil Modulus Multiplier(M) on Maximum Wall Movement and Surface Settlement . . . . .	43
3.18	Effect of Strut Stiffness on Maximum Wall Movement and Surface Settlement . . . . .	44

3.19 Effect of Strut Stiffness on Maximum Wall Movement and Surface Settlement . . . . .	44
3.20 Effect of Wall Stiffness on Maximum Wall Movement and Surface Settlement . . . . .	45
3.21 Effect of Wall Stiffness on Maximum Wall Movement and Surface Settlement . . . . .	45
3.22 Effect of Surcharge Load on Maximum Wall Movement and Surface Settlement . . . . .	46
3.23 Effect of Surcharge Load on Maximum Wall Movement and Surface Settlement . . . . .	46
3.24 Effect of Strut Spacing on Maximum Wall Movement and Surface Settlement . . . . .	47
3.25 Effect of Strut Spacing on Maximum Wall Movement and Surface Settlement . . . . .	47
3.26 Effect of L/B Ratio on Maximum Wall Movement and Surface Settlement	48

# List of Tables

3.1	Basic Conditions and Variations for Parametric Study of Structed Excavations . . . . .	30
3.2	Comparison of Absolute Values of Displacements . . . . .	40

# Nomenclature

$B$	Width of excavation
$[B]$	Strain-displacement matrix
$[D]$	Elasticity matrix
$E$	Elastic modulus
$E_o$	Elastic modulus of soil at ground level
$E_y$	Elastic modulus of soil at depth $y$ meter from top
$EI/h^4\gamma$	Normalized wall stiffness
$\{F^e\}$	Force vector
$\{f^e\}$	Body force on element
$f_i$	Body force acting at node $i$
$h_e$	Thickness of element
$[J]$	Jacobian matrix
$\  J \ $	Determinant of jacobian
$[K^e]$	Elemental stiffness matrix
$L$	Length of excavation
$[L]$	Differential operator matrix
$m_e$	Rate of variation of Elastic modulus with depth
$M$	modulus multiplier, $E = MS_u$
$N_1, N_2, \dots$	Shape functions
$[N]$	Shape function matrix
$\{Q^e\}$	Load vector
$S = AE/l$	Strut stiffness
$S^e$	boundary
$S_u$	Undrained shear strength
$S/h\gamma$	Normalized strut stiffness
$t_i$	Boundary traction at node $i$
$\{u\}$	Displacement vector
$\{u_i\}$	Nodal displacement vector
$V^e$	Volume of the element
$[x]$	Coordinate vector

$\sigma_{ij}$	Stress component
$\epsilon_{ij}$	Strain component
$\delta$	Variational operator
$\alpha$	Normalised displacement
$\nu$	Poissons ratio
$[\Delta]$	Nodal displacement matrix
$\Omega^e$	Volume /Area of element
$\Omega$	Volume /Area domain
$\xi, \eta, \zeta$	Natural coordinates
$\Gamma$	Surface

# Chapter 1

## *Introduction*

### 1.1 General

Deep excavations become necessary for foundations of high rise buildings, which are quite common owing to the rapid urbanization. Excavations also play vital role in the construction of tunnels, subway stations, industrial substructures. Excavations are dominantly used particularly in congested cities to provide under ground space. Very often, the excavations are carried out in close proximity of existing buildings and infrastructures, which have been earmarked for preservation, or high rise buildings. Both types of structures are sensitive to ground movements. Some infrastructures, such as Mass Rapid Transit (MRT) tunnels and tracks , have very low tolerance for ground movements. Controlling ground surface settlement around the excavation zone is an essential task in the design of excavation. Excessive ground settlement frequently damages the adjacent properties in urban areas. It is a legal necessity with any new construction to provide protection to the adjacent structures when excavation is carried to an appreciable depth. Without adequate lateral support the excavation will almost certainly cause loss of bearing capacity, settlements, or lateral movements to existing structures.

Excavations in built-up areas require reliable support systems to prevent collapse and limit the adjacent ground movements. It is important to design supporting system for safe, economical, and serviceable excavations. The basic functions of excavation support systems are to provide stability and minimize movements of the adjacent ground. Methods for predicting system stability are reasonably well established; however , this is not so in the case for movements. The movements of the supporting system and the adjacent ground are, in fact, a more involved problem than stability, since movements are influenced by more factors than stability. The factors which are mainly influencing the excavations are soil properties, structural properties of the supporting system,

construction sequence, workmanship, and excavation geometry. In nature it is difficult to assess the behavior of soil. The presence of supporting system makes it even more complex with various support system. The exact behavior of the excavation from classical theories of soil mechanics is difficult to obtain.

## 1.2 Literature Review

The available literature on deep excavations have been reviewed under the following categories.

1. Analytical methods.
2. Observational data from case studies
3. Case studies using reinforcing techniques
4. Case studies using effective-stress path approach.
5. Studies on 3-D modeling

### 1.2.1 Analytical Methods.

Palmer and Kenney (1971) presented the results of an analytical study of a typical braced excavation in weak clay with the sheet-pile wall extending to bedrock. They studied the relative influences on the behavior of the excavation of such factors viz., soil strength and deformation characteristics, stiffness of the wall and of the bracing system, etc. The simulation of an actual case history has also be presented to indicate the applicability of their computer program. They found that the vertical spacing of struts mainly affect the magnitude of strut load but not of pile deflection and the small amount of pre-stress could be beneficial in ensuring early effectiveness of the supporting system but that a large amount of pre-stress may not provide additional benefits.

Mana and Clough (1981) developed a predictive methodology to estimate maximum lateral wall deflections and soil settlements for cross lot braced systems in soft to medium clay deposits. The method is based upon behavior measured in the field, and supplemented with results from finite element analysis developed to closely simulate the field response of braced excavation system. They have developed a correlation between movements and the potential for basal heave which is defined as the factor of safety against basal heave (Terzaghi. 1959). Factor of safety against basal heave is calculated using Terzaghi's approach. They found that the rate of magnitude of movements increases rapidly as the factor of safety approaches to one. They presented nondimensional charts for estimating the wall movements and surface settlements as a function of factor of safety, strut stiffness, wall stiffness, depth of firm layer, excavation width and strut preload.

Clough and Hansen (1981) have studied the significance of clay anisotropy on the braced wall behavior. They reported a steady rate of decrease in the undrained shear strength as the angle of reorientation increases. In their analysis they modified the basal heave failure mechanism proposed by Terzaghi to incorporate the anisotropic strength effects. They have done parametric studies using finite element analysis using nonlinear elastic soil model for the plain strain conditions. They found that the lateral wall movements and ground surface settlements associated with anisotropic soil are always larger than those of isotropic soil and the difference becomes larger as the depth of excavation increases.

Dysli and Fontana (1982) analyzed two very deep excavations in clayey soil and compared the results with many in-situ observations. They have used finite element analysis with nonlinear constitutive laws. Both the excavations were laterally supported by slurry walls. In the analysis they modeled the soil as elastic perfectly plastic material in their analysis.

Matos Fernandes (1985) have studied a deep excavation for construction of underground large building. The excavation was supported multi-strutted diaphragm wall. With the help of finite element model several analyses have been carried out at different stages of the work and the results are compared with the field observations. They have shown that there is remarkable difference between the predicted and measured values of the wall displacements, and this is attributed to the difference in effective and theoretical stiffness of the considered strut. The effective stiffness of strut which is evaluated based on the results of the observation, takes the average value of strut load in the stage next to its placement.

Hata et al. (1985) presented the performance of anchored diaphragm wall by adapting finite element technique based on elasto plastic constitutive equations together with an algorithm. They also analyzed the seepage flow during and after the excavation. The results from their model have shown good agreement with the monitored behavior of the excavation. One of the Major conclusions of their analysis was that the displacements of the diaphragm wall due to increase of excavation depth do not depend much on the passive earth pressure produced by the clay below the level of excavation but depends on bracings, struts, and earth anchors.

Wong et al. (1989) presented a simplified method to estimate the lateral deflections of braced sheet pile walls in soft clay. They examined the influence of a number of factors that affect the lateral wall displacement, settlement, excavation width, and heave of deep excavations. They used EXCAV (Duncan and Chang 1977) computer program for their analysis and nonlinear hyperbolic stress-strain relationship has been used to describe the behavior of the soil. Computed values have matched well with results from finite element analysis and a number of case records. The analysis shows that the set-

tlements, lateral displacements, and heave are mainly controlled by the factor of safety with respect to basal heave. Also, the stiffness of the sheet pile or the diaphragm wall is important. They found from the analysis that the lateral wall deflection increases almost proportionally with the width of excavation and the deflection ratio increases linearly with the excavation depth.

Lee et al. (1989) compared the results obtained from the finite element analysis package CRISP with those obtained from the instrumented, struttured sheet pile excavation. The lateral sheet pile wall displacement profile is similar in trend with the measured profiles at the site. They have found that CRISP was under predicting the maximum lateral wall movements and basal heave.

Borja (1989) developed a numerical procedure for simulating the excavation in cohesive soils incorporating time dependent creep effects. The excavation analysis algorithm incorporated creep effects in the constitutive model.

Borja (1990) developed an accurate, stable, efficient algorithm for analyzing the problem of nonlinear incremental excavation employing critical state theory to predict the deformation behavior of braced excavations in mud deposits. Finite element analysis with a new stress point integration algorithm was proposed for the cam-clay model of critical state soil mechanics. The proposed model has applied to study various cases of 2-dimensional and 3-dimensional problems. For the field application, a case study of deep excavation was studied and the results were in good agreement with the measured values.

Finno et al. (1991) has presented the simulations of construction of 40-ft-deep excavation in saturated clays using a coupled finite element formulation. Surface and subsurface ground movements, pore water pressures, and sheet-pile deflections were measured throughout the construction at the site and were compared with the computed sheet-pile deformations. They found that the computed deformations agreed quite well with observed data at all stages. They also found that the computed ground movements quickly diverged from observed responses when relatively large strains are included in the soil mass and result in localized strains. These strains correspond to the peak shear stress.

Whittle et al. (1993) described the applicability of a finite element analysis for modeling the top-down construction method of deep excavation. Analysis incorporates coupled flow and deformations within the soil for real time simulation of construction activities; a numerically accurate algorithm for excavation in nonlinear soil; and advanced constitutive modeling of clay behavior. Predictions are evaluated through comparisons with extensive field data, including wall deflections, soil deformations, and surface settlements. Difference between predicted and measured wall movements were attributed primarily to post construction shrinkage of the roof and floor system.

A modified base-case analysis, incorporating these factors, greatly improves agreement with the measured data. The results demonstrate that reliable and consistent predictions of soil deformations and ground water flow can be achieved by advanced methods of analysis.

Hashash et al. (1996) presented the results of a series of numerical experiments, using nonlinear finite-element analysis, which investigate the effect of wall embedded at a depth, support conditions, and stress history profile around the braced diaphragm wall in a deep clay deposit. In their analysis they used comprehensive effective stress soil model (MIT-E3) to describe the important aspects of clay behavior, including small strain nonlinearity and anisotropic stress-strain strength. The results obtained from the numerical experiments have important practical implications for the estimation and control of ground movements for proposed deep excavations in Boston. They found that the wall length has a minimal effect on the prefailure deformations for excavations in deep layers of clay, and has no beneficial influence on the wall embedded.

Bose et al. (1997) presented a parametric study of a typical instrumented section of the Calcutta Metro Construction, Finite element method has been used to simulate the sequential process of excavation. Soil nonlinearity was modeled using modified Cam-clay constitutive relationship, while the diaphragm wall was assumed to be linear. The wall thickness was varied to alter the rigidity of the braced system and its effect on the performance of the braced excavation was investigated. They found that the increase in wall thickness beyond that provided in the field hardly influences the wall deflection and ground settlement; and the total strut forces increases due to redistribution of horizontal stress in the vicinity of excavation.

Ou et al. (2000) presented the building responses and ground movements induced by an excavation using the top down construction method. The ground surface settlements and lateral wall deflections for each stage were observed. The displacement performance of the soil was studied by examining the strain field and displacement vector developing in soil mass behind the wall. They found that the maximum vertical soil movement does not occur on the ground surface but occurs at a certain depth below the ground surface and most of the soil behind the wall has non-zero volume change during excavation. It may be due to the consolidation and soil creep behavior contribution. Several factors affect the building performance during excavation such as, type of foundation, length of excavation, and size and shape of the foundation. A building with a mat foundation near a relatively short excavation side should be subjected to only a slight inclination, but may experience a larger inclination if the excavation side is relatively long.

### 1.2.2 Observational Data from Case Studies

Thomas D.O'Rourke (1981) examined the ground with braced excavations with emphasis on the relationship between ground movements and various aspects of the construction process. Sources of lost ground are reviewed and deformation patterns at the excavation site are related to displacement patterns at the ground surface adjoining the excavation. Case histories of braced cuts in both sands and soft clays are summarized and recommendations for ground movement control are made on the basis of observational data. It has been found from the observations and model test data that a direct relationship between wall deformation and the ratio of horizontal movement of settlement adjacent to the cut exists. As cantilever-type movements are allowed to dominate during construction, the proportion of horizontal to vertical movement increases.

Clough and Reed (1984) presented the results of an instrumentation program for monitoring the behavior of the excavation. The excavation was 7.6m wide and 9.2m deep. Because of the low strength of the soil they found large movements developing at the site during excavation. They presented the profiles of lateral wall movements at different cross sections of the excavations, measured strut loads and nondimensional plots against lateral wall movements.

Massad (1985) carried out a field research on braced excavations in laterite and weathered sedimentary soils. The effect of temperature on strut loads is studied, and a comparison of lateral displacements and apparent earth pressures is made with the soils from other origins. He found that the apparent earth pressure diagram has to be modified incorporating the effects caused by thermal dilatation of the struts. The apparent pressure envelope revealed that the maximum value of  $0.8\gamma H$ , smaller than it would be expected if compared to stiff sedimentary soils from other origins. Another conclusion is that temperature effect is relevant for the design.

El Nanhas et al. (1989) presented the geotechnical in-situ behavior of the braced walls based on the results of the monitoring instrumentation program. The effect of bracing systems, wall stiffness and soil properties on magnitude and shape of the measured soil deformation were discussed.

### 1.2.3 Case studies using Reinforcing Techniques

Shen et al. (1981) described the construction of a relatively new earth support system for deep excavations based on the concept of soil reinforcement. The system used the in situ earth reinforcement technique to strengthen the native soil, and is different from the conventional systems that serve to retain soils behind a vertical cut. The field results were compared with the predictions obtained from the finite element method

of analysis. The agreement between measured and the predicted movements was quite good.

#### 1.2.4 Case Studies using Effective-Stress Path Approach.

Tominga et al. (1985) reported the application of stress path method to excavations. With adoption of stress path approach proposed by Lambe, a simple method was proposed to evaluate the soil properties. This model was applied to the case study of deep excavation, predictions were made and compared with the field measurements. They also concluded that the analysis considering stress path method is giving results with higher accuracy when compared with the conventional elastic spring beam analysis.

Charles (1999) presented the total and effective-stress variations adjacent to a diaphragm wall during construction of a 10m deep excavation in stiff fissured clays. Effective-stress changes associated with the vertical and horizontal stress relief during the excavation were obtained from these measurements and expressed in terms of stress paths. They found that the field effective-stress paths in front of the wall were similar to those stress paths observed in the UE tests in the laboratory; however, for the stress paths behind the wall and field observations did not correspond well with laboratory undrained tests on natural specimen in compression, except when stress state approached active failure.

#### 1.2.5 Studies on 3-D Modeling

Chang-Yu Ou et al. (1996) developed a nonlinear, three dimensional finite element technique for deep excavation. They studied the effect at the corner on the deflection behavior of an excavation in a medium clayey subsoil stratum. In their analysis, the excavation wall was assumed as a liner-elastic material, and soil was assumed to behave as an elasto-plastic material described by hyperbolic model. They performed a series of parametric studies, and a tentative relationship for estimating the 3-D maximum wall deflection using 2-D finite element analysis is proposed. They concluded that short primary walls were affected by the existence of the corner while in the longer primary walls, most of the sections were in plain strain condition.

Fook-Hou Lee et al. (1998) discussed the effects of corners on wall deflection and ground movement around multistrutted deep excavations. The field data were back-analyzed using 2-D and 3-D finite element analysis to study the ability of both types of analysis in predicting the observed behavior. They found that the significant differences between field data and the numerical predictions indicates that some of the factors have not been accounted for. Some of these are, construction delay, over excavation etc.,

But it is very difficult to account these factors in analysis. They concluded that the 3-D analysis predict wall deflections better than the 2-D analysis.

Kameswara Sharma (1997) presented three dimensional analysis of deep excavations, finite element package CRISP90 has been used for his analysis. Three dimensional analysis is carried out and the results are compared with the two dimensional plane strain analysis results. Effect of various parameters like soil elastic modulus, strut stiffness, wall stiffness and length to width ratio have been studied. From the results he concluded that small movements are obtained from three dimensional analysis than two dimensional plane strain analysis.

### 1.3 Scope of Present Investigation

Based on the brief literature review carried out, it has been founded that most of the work pertaining to the analysis of deep excavations has been done considering the plain strain conditions at the site. A few have presented the corner effects, effect of embedded length of wall in to the ground in 3-dimensional modeling and compared their results with 2 dimensional plain strain results. But they don't studied the other important parameters which are mostly influencing the system of deep excavation.

The main object of present study was to carry out the various types of parametric studies of the deep excavations in 2-dimensional plane strain conditions and compared with the existing results reported in literature by Mana and Clough (1981). The same parametric studies were carried out with the 3-dimensional modeling and compared with the results of plane strain.

The study was undertaken in the following manner.

- Parametric studies has been carried out for two dimensional plane strain analysis of the excavation. The results have compared with the existing results reported in literature by Mana and Clough (1981). The soil and wall were modeled as elastic material and the struts were modeled as two noded spring elements. The geometric nonlinearity has been considered in the analysis.
- Three dimensional analysis of excavation has been modeled with elastic material and the results obtained from the analysis have been compared with the two dimensional plane strain results.
- The effect of significant parameters on wall movements and surface settlements have been also carried out. Soil with elastic modulus varying linearly with depth has also been considered.

## 1.4 Organization of the Thesis.

Present work has been organized in the following manner.

In Chapter 2, the basic principle of finite element analysis has been described. Formulation of two dimensional plane strain case and three dimensional finite element analysis is presented. Two dimensional eight noded quadrilateral element, two noded spring element and three dimensional twenty noded solid elements were used in the analysis. The details of the finite element package NASTRAN and solution employed in the package are presented.

In Chapter 3, the results and discussions are presented. The problem of excavation initially modeled with 2-dimensional plane strain analysis has carried out. Parametric studies were carried out with the same properties of the excavation. The analysis has been carried out for three dimensional model with  $L/B=1.0$ . The soil and sheet pile wall is modeled as elastic material and the struts are modeled as spring elements. The elastic modulus of soil is taken as the linearly varying with depth. In two dimensional analysis, the effect of parameters like soil modulus, wall stiffness, strut stiffness, effect of excavation width, surcharge loads, depth of water table, strut spacing on maximum lateral wall movements and maximum surface settlements have been studied. In the three dimensional analysis the same parametric studies were carried out and finally compared with the two dimensional plane strain results.

In Chapter 4, the conclusions drawn from the study and the scope for possible extensions are indicated.

# Chapter 2

## *Formulation*

### 2.1 General

The nature of soil is highly complex and it is extremely difficult to assess the behavior of soil in the field. When excavations are considered, it is even more complex as the behavior of adjacent ground is influenced by various parameters like soil properties, supporting systems parameters such as wall stiffness, strut stiffness, strut spacing, geometric parameters of excavation such as excavation width, depth of firm layer on which the wall rests. It is highly difficult to analyse the adjacent ground and supporting system from the classical methods of soil mechanics due to complexity of the soil properties and the complex nature of supporting system of the excavation. Finite element method can be used in the analysis of such systems by simulating the conditions existing in the field. In this section, basic formulation for the finite element analysis, different elements used in the analysis and the method followed for simulation are discussed.

### 2.2 Finite Element Method

Most of the physical phenomena involves formulation of the process and numerical analysis of mathematical model. The formulation results in mathematical statements, often differential equations relating quantities in the physical process. Their derivation for most of the problems is not unduly difficult but the solution by exact analysis is a formidable task. In such cases approximate methods of analysis provide alternate means of finding solutions. Among these, finite difference method and the variational methods such as Rayleigh Ritz method and Galarkin's method are most frequently used. But their suitability is only limited for simplified geometry.

It is possible to model the spatial distribution of complex boundary shapes or phys-

ical properties using finite element method. Furthermore, arbitrary boundary conditions can be likewise to be taken into consideration, such as displacement constraints and/or stress constraints. Such a wide applicability is one of the main reasons for the popularity of the finite element method.

The method is characterized by the following features.

- The domain of the problem is represented by a collection of simple sub-domains, called finite elements. The collection of finite elements is called finite element mesh.
- Over each finite element, the physical process is approximated by functions of desired type (polynomials or otherwise), and algebraic equations relating physical quantities at selective points, called nodes, of the element are developed .
- The equations are assembled using continuity of physical quantities.

Besides the method with the above features there can be more than one finite element models developing on the differential equations and methods used to derive the algebraic equations. Rayleigh Ritz method and polynomial approximations are generally used to construct the finite element quantities.

## 2.3 Formulation

In this section the basic formulation of finite element method is discussed. Static formulation of finite element method can be equivalently be derived from the virtual work principle and variational principle. “The principle of virtual work states that a body is in equilibrium under the action of forces for arbitrary virtual displacement, form a state of compatible deformation with compatible strain, the virtual work is equal to the virtual strain energy” which can be represented as,

$$\int_{V^e} (\sigma_{ij} \delta \epsilon_{ij}) dv - \int_{V^e} f_i \delta u_i dv - \int_{S^e} t_i \delta u_i ds = 0 \quad (2.1)$$

Where  $v^e$  denotes the volume of the element  $e$  and  $s^e$  the boundary.  $\delta$  denotes the variational operator.  $\sigma_{ij}$  and  $\epsilon_{ij}$  are the components of the stress and strain tensors and  $f_i$  and  $t_i$  are the components of the body force and boundary traction respectively.

The first term in equation 2.1 corresponds to the virtual strain energy stored in the body, the second represents the virtual work done by the body forces, and the third

represents the virtual work done by surface tractions. Rewriting the above equation, we get

$$\begin{aligned}
 & \int_{\Omega^e} (\sigma_x \delta \epsilon_x + \sigma_y \delta \epsilon_y + \sigma_z \delta \epsilon_z + \sigma_{xy} \delta \epsilon_{xy} + \sigma_{yz} \delta \epsilon_{yz} + \sigma_{xz} \delta \epsilon_{xz}) dx dy dz \\
 & - \int_{\Omega^e} (f_x \delta u_x + f_y \delta u_y + f_z \delta u_z) dx dy dz \\
 & - \oint_{\Gamma^e} (t_x \delta u_x + t_y \delta u_y + t_z \delta u_z) ds = 0
 \end{aligned} \tag{2.2}$$

Where  $f_x$  and  $f_y$  are body forces per unit volume, and  $t_x$  and  $t_y$  are the tractions per unit area. When stresses are expressed in terms of strains and strains in terms of displacements, then the equation 2.1 becomes based on the principle of minimum potential energy.

Approximating  $u$  over  $\Omega^e$  by the finite element interpolations, the displacements and strains are, as follows

$$\begin{aligned}
 \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} &= \begin{Bmatrix} \sum_{j=1}^n u_i N_i \\ \sum_{j=1}^n v_i N_i \\ \sum_{j=1}^n w_i N_i \end{Bmatrix} \\
 &= \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 & \dots & N_n & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 & \dots & 0 & N_n & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 & \dots & 0 & 0 & N_n \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ w_1 \\ u_2 \\ v_2 \\ w_2 \\ \vdots \\ u_n \\ v_n \\ w_n \end{Bmatrix}
 \end{aligned} \tag{2.3}$$

$$\{U^e\} = [N^e] \{\Delta^e\} \tag{2.4}$$

we have

$$\left. \begin{array}{l} \{\epsilon^e\} = [B^e] \{\Delta^e\} \\ \{\sigma^e\} = [D] [B^e] \{\Delta^e\} \end{array} \right\} \quad (2.5)$$

where

$$[B^e] = [L] [N] \quad (2.6)$$

and  $[L]$  is the matrix differential operator defined as

$$[L] = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \end{bmatrix} \quad (2.7)$$

we have

$$\left. \begin{array}{l} \begin{array}{l} \{\delta u\} \\ \{\delta v\} \\ \{\delta w\} \end{array} = [N] \{\delta \Delta^e\} \\ \{\delta \epsilon\} = [B] [\delta \Delta^e] \end{array} \right\} \quad (2.8)$$

Substituting these expressions for displacements and strains in equation 2.2 and writing it in the matrix notation results in

$$0 = \int_{\Omega^e} \{\delta \Delta^e\}^T \left( [B^e]^T [C] [B^e] \{\Delta^e\} \right) dx dy dz - \int_{\Omega^e} \{\delta \Delta^e\}^T [N]^T \begin{Bmatrix} f_x \\ f_y \\ f_z \end{Bmatrix} dx dy dz - \oint_{\Gamma^e} \{\delta \Delta^e\}^T [N]^T \begin{Bmatrix} t_x \\ t_y \\ t_z \end{Bmatrix} ds \quad (2.9)$$

i.e,

$$\{\delta \Delta^e\}^T ([K^e] \{\Delta^e\} - \{f^e\} - \{Q^e\}) = 0 \quad (2.10)$$

Since the equation holds for any arbitrary variations  $\{\delta \Delta^e\}$ , the expression in the parenthesis should be identically zero, giving

$$[K^e] \{\Delta^e\} = \{f^e\} + \{Q^e\} \quad (2.11)$$

where

$$[K^e] = \int_{\Omega^e} [B^e]^T [D] [B^e] dx dy dz \quad (2.12)$$

$$\{f^e\} = \int_{\Omega^e} [N]^T \begin{Bmatrix} f_x \\ f_y \\ f_z \end{Bmatrix} \quad (2.13)$$

$$[Q^e] = \oint_{\Gamma^e} [N]^T \begin{Bmatrix} t_x \\ t_y \\ t_z \end{Bmatrix} \quad (2.14)$$

In the above equations  $[K^e]$  is the element stiffness matrix of order  $3n \times 3n$  and  $\{F^e\} = \{f^e\} + \{Q^e\}$  is the element force vector of order  $3n \times 1$ .

Similarly two dimensional finite element equations can be derived for plane strain case in x-y plane by letting the strain in direction z to zero.

## 2.4 Finite Element Model

### 2.4.1 Two Dimensional Model

Eight noded quadrilateral elements are used for the descretization of the soil medium and wall sections within the excavation domain, and two noded spring elements are used for modeling of struts. The boundary at the bottom of the excavation and the free ends of the struts are restrained both in x and y directions. The boundary at the side away from the excavation and the side of symmetry of excavation both are restrained in x-direction. Because of symmetry in the configuration of excavation geometry, half of the portion of excavation was considered for the analysis. The model is shown in Fig. 2.1

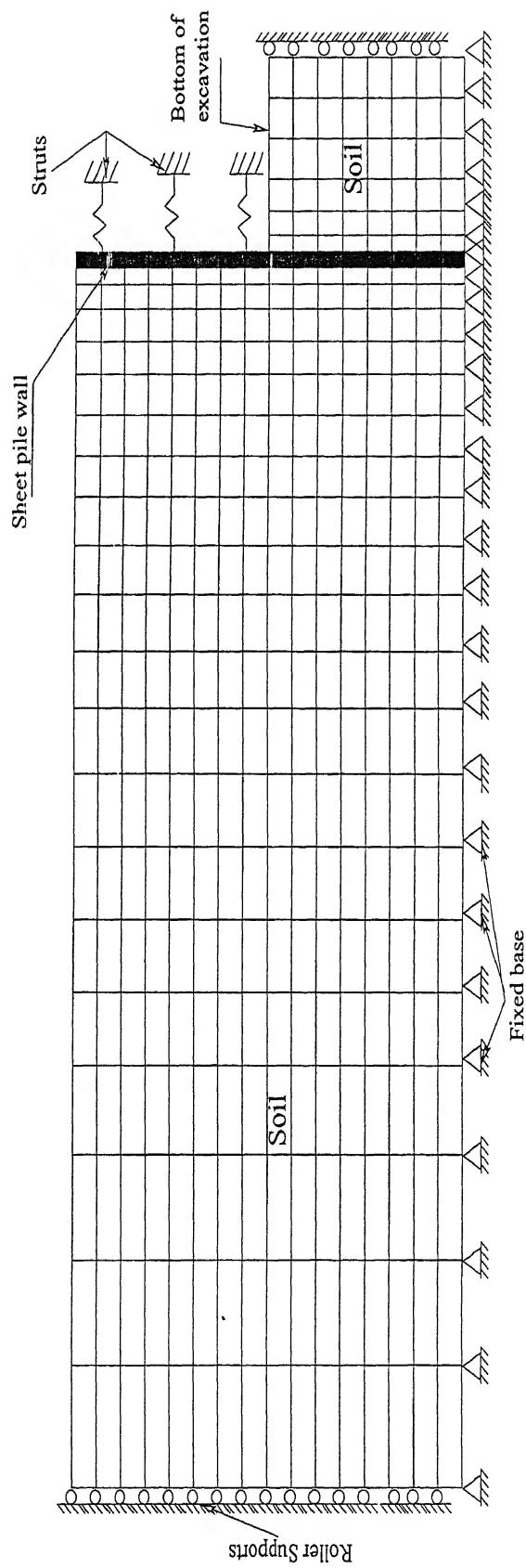


Figure 2.1: Two Dimensional Model with Descretization

## Element Stiffness Matrix

A typical eight noded quadrilateral element and 2 noded spring elements are shown in Fig. 2.2

The shape function for the eight noded quadrilateral element can be given as follows.

For corner nodes

$$N_i = \frac{1}{4} (1 + \xi^2) (1 + \eta) (\xi + \eta - 1) \quad (2.15)$$

For mid side nodes

$$\xi_i = 0$$

$$N_i = \frac{1}{2} (1 - \xi^2) (1 + \eta) \quad (2.16)$$

$$\eta_i = 0$$

$$N_i = \frac{1}{2} (1 + \xi) (1 - \eta^2) \quad (2.17)$$

The elasticity matrix for plane strain case can be expressed as,

$$[D] = \frac{E (1 - \nu)}{(1 + \nu) (1 - 2\nu)} \begin{bmatrix} 1 & \nu / (1 - \nu) & 0 \\ \nu / (1 - \nu) & 1 & 0 \\ 0 & 0 & (1 - 2\nu) / 2 (1 - \nu) \end{bmatrix} \quad (2.18)$$

The coordinate transformation matrix (Jacobian)  $J$  is,

$$J = \begin{bmatrix} \sum \frac{\partial N_i}{\partial \xi} x_i & \sum \frac{\partial N_i}{\partial \xi} y_i \\ \sum \frac{\partial N_i}{\partial \eta} x_i & \sum \frac{\partial N_i}{\partial \eta} y_i \end{bmatrix} \quad (2.19)$$

In the above expression the summation is for  $i = 1$  to  $i = 8$

The stiffness matrix for plane strain elasticity can be expressed as,

$$[K^e] = h_e \int_{\Omega^e} [B^e]^T [D] [B^e] dx dy \quad (2.20)$$

In the above equation  $h_e$  is the thickness. Approximating the displacements over the domain using the shape functions and converting the displacements from global to local coordinate system by using Jacobian of transformation, the stiffness matrix  $[K^e]$  can be written as,

$$[K^e] = h_e \int_{-1}^{+1} \int_{-1}^{+1} [B]^T [D] [B] \| J \| d\eta d\xi \quad (2.21)$$

Where  $\| J \|$  is the determinant of the Jacobian matrix.

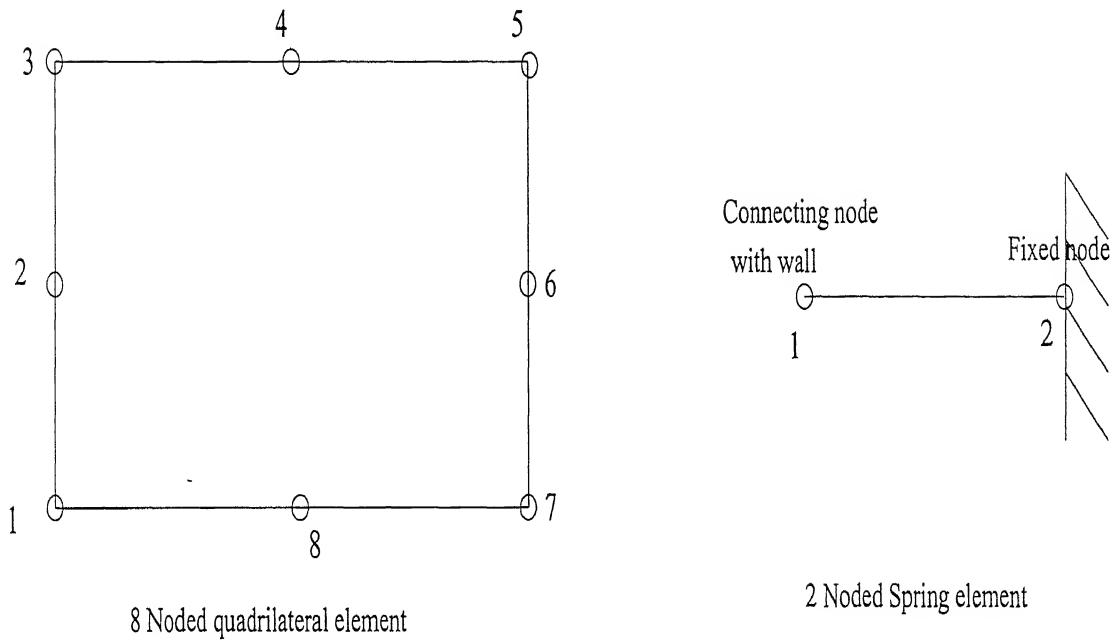


Figure 2.2: Eight Noded Quadrilateral Element and Two Noded Spring Element

## 2.4.2 Three Dimensional Model

A twenty noded hexagonal isoparametric element is considered as the basic element in modeling the problem for three dimensional analysis. Both the soil and the wall are modeled using the above mentioned basic element, where as struts are modeled using the one dimensional two noded spring elements. The free node of each of the spring element is fixed and the other node is connected to the corresponding node on the wall. The vertical boundaries in all the sides are restrained laterally and bottom of the model is restrained in all three directions. The details of the discretization are shown in Fig. 2.3. The element stiffness matrix equation 2.12 can be derived as described in following section.

### Element Stiffness Matrix

A typical twenty noded element is shown in Fig. 2.4 in local coordinate system  $\xi, \eta, \zeta$  and as well as global coordinate system  $x, y, z$ . The coordinate mapping which relates the local and global coordinate systems can be expressed as

$$X = [N_i] \{X_i\} \quad (2.22)$$

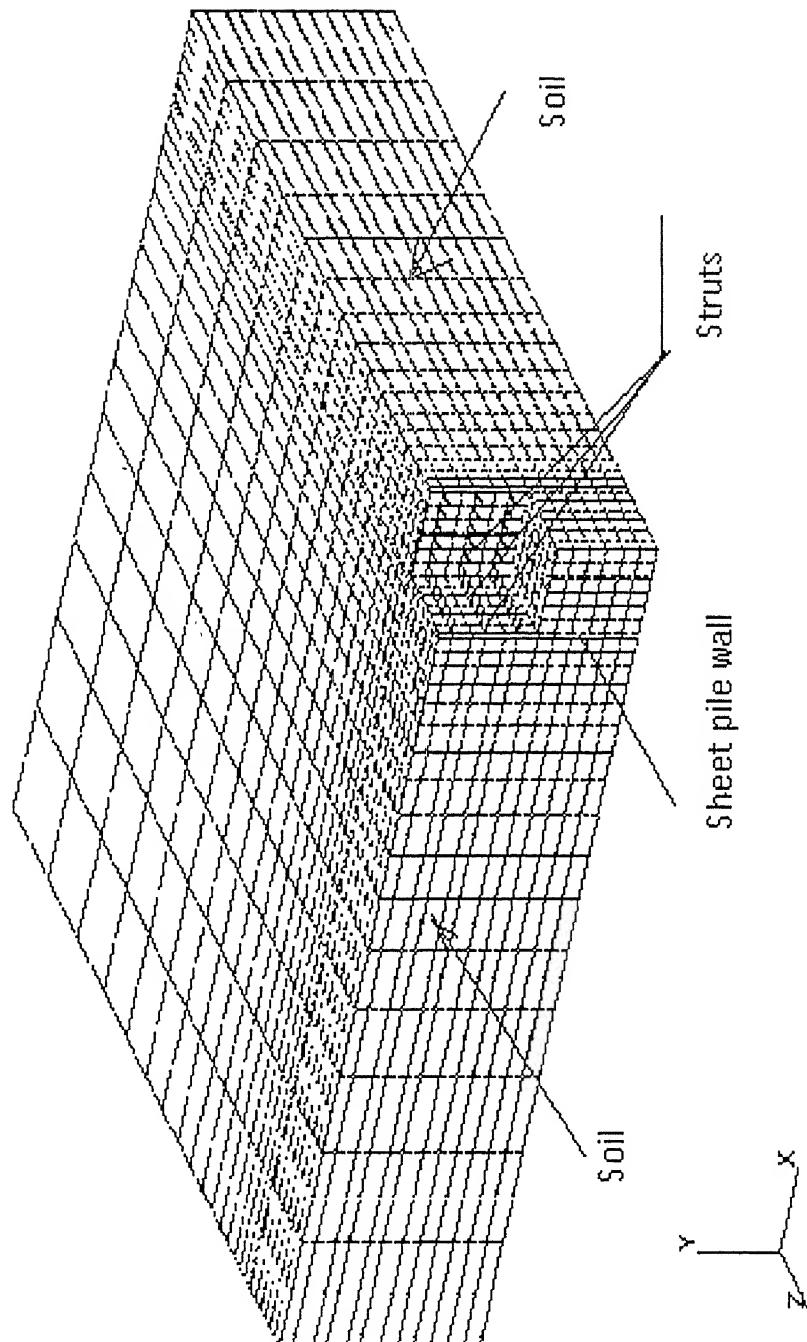


Figure 2.3: 3-D model with descritisation

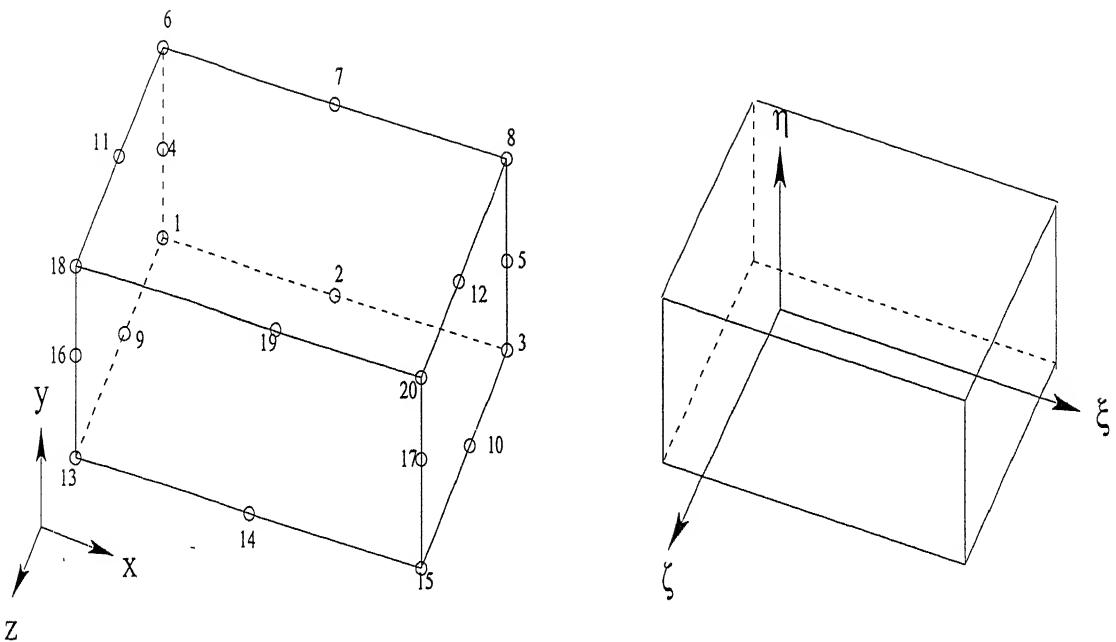


Figure 2.4: Twenty Noded Brick Element with Coordinate System

where  $X$  and  $X_i$  are the vectors of independent variables, and coordinates for the node  $i$  respectively;  $N_i$  is the sub matrix of order equal to the number of space dimensions. In the above equation the subscripts vary from 1 to 20, and

$$X = \begin{bmatrix} x & y & z \end{bmatrix}^T \quad (2.23)$$

$$[N_i] = \begin{bmatrix} N_i & 0 & 0 \\ 0 & N_i & 0 \\ 0 & 0 & N_i \end{bmatrix} \quad (2.24)$$

$$X_i = \begin{bmatrix} x_i & y_i & z_i \end{bmatrix}^T \quad (2.25)$$

The mapping function (Zienkiewicz 1979) in terms of three normalized coordinates for twenty noded quadratic element is given as follows.

For corner nodes

$$N_i = \frac{1}{8} (1 + \xi) (1 + \eta) (1 + \zeta) (\xi + \eta + \zeta - 2) \quad (2.26)$$

and for typical mid side node ( $\xi_i = 0, \eta_i = 0, \zeta_i = 0$ )

$$N_i = \frac{1}{4} (1 - \xi^2) (1 + \eta) (1 + \zeta) \quad (2.27)$$

The displacement vector  $u$  can be expressed in terms of its nodal values using

shape function, which is expressed in the local coordinate system of the element. In the formulation, the shape functions are same as the coordinate mapping functions of the element. Hence,

$$\{u\} = [N_i] \{\Delta^e\} \quad (2.28)$$

This can be valid for elemental quantities, as expressed in equations 2.4, 2.5, and 2.6. From equation 2.6 and the differential operator  $[T]$  expressed in equation 2.7, the integrand in the stiffness matrix from equation 2.12 contains the terms in global coordinate system. The integral is expressed in terms of local coordinates by using Jacobian of transformations. The Jacobian of transformation matrix can be expressed as,

$$J = \begin{bmatrix} \sum \frac{\partial N_i}{\partial \xi} x_i & \sum \frac{\partial N_i}{\partial \xi} y_i & \sum \frac{\partial N_i}{\partial \xi} z_i \\ \sum \frac{\partial N_i}{\partial \eta} x_i & \sum \frac{\partial N_i}{\partial \eta} y_i & \sum \frac{\partial N_i}{\partial \eta} z_i \\ \sum \frac{\partial N_i}{\partial \zeta} x_i & \sum \frac{\partial N_i}{\partial \zeta} y_i & \sum \frac{\partial N_i}{\partial \zeta} z_i \end{bmatrix} \quad (2.29)$$

In the above expression  $\sum$  indicates the summation over the number of nodes of the element i.e. 20

The elasticity matrix  $[D]$  for three dimensional solid can be expressed as,

$$[D] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \quad (2.30)$$

The stiffness matrix of the element now can be expressed as,

$$[K^e] = \int_{-1}^{+1} \int_{-1}^{+1} \int_{-1}^{+1} [B]^T [D] [B] \| J \| d\zeta d\eta d\xi \quad (2.31)$$

where  $\| J \|$  indicates the determinant of the Jacobian matrix  $J$ .

### 2.4.3 Variation of Elastic Modulus with Depth

Most of the soils in nature don't have constant elastic modulus. The elastic modulus varies with depth. The variation of elastic modulus is needed to be incorporated to simulate the actual field condition. The soil medium with linearly varying elastic

modulus has been considered in the analysis and profile of variation as shown in Fig. 2.5. The variation of elastic modulus with depth can be expressed in the forms,

$$E_y = E_o + m_e y \quad (2.32)$$

where  $E_y$  is elastic modulus of soil at a depth of  $y$  from the top

$E_o$  is elastic modulus of soil at ground level

$m_e$  is the rate at which the elastic modulus is increases with depth

and  $y$  is the depth at which the elastic modulus of soil is to be found.

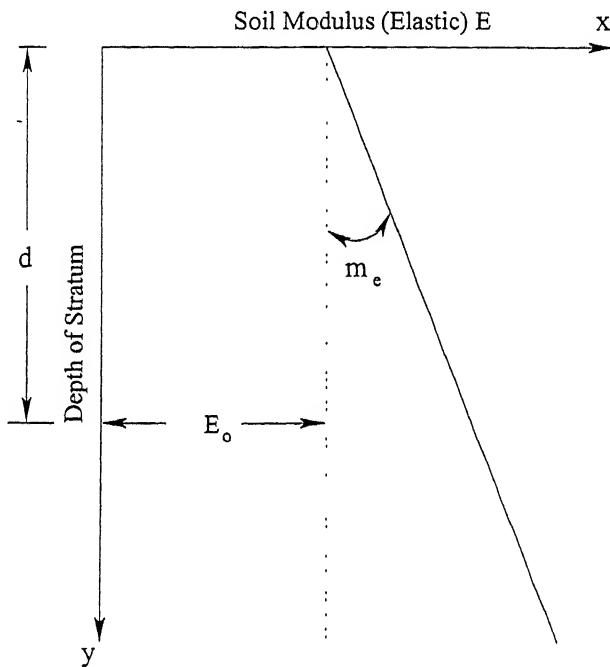


Figure 2.5: Elastic Modulus of Soil Linearly Varying with Depth

#### 2.4.4 Details of Package used

In this section a brief details of the finite element package (NASTRAN) used for the analysis of deep excavations are presented.

The computer package NASTRAN is used for analysing the present problem. NASTRAN is one of the products of the MSC.Software Corporation (NYSE: MNS). The company was founded by Dr. Richard MacNeal and Mr. Robert Schwendler in 1963 and the first version of NASTRAN was released in 1971.

The various analysing capabilities associated with the package are summarized below.

### Types of Analysis:

- Linear static, Nonlinear static, Transient response, Hyper elastic Frequency response, and Nonlinear transient analysis of both two dimensional and three dimensional models.

### Properties of Models:

- Linear elastic, Nonlinear elastic, Hyper elastic, Elastoplastic, Failure and Creep properties in all the cases of isotropic, anisotropic and composite materials.

### Element Types:

- Triangular elements of 3, 4, 6, 7, 9, 13 noded, Quadrilateral elements of 4, 5, 8, 9, 12, 16 noded in two dimensional models and wedge elements of 6, 7, 15, 20, 21, 24, 52 noded and 8, 9, 20, 21, 26, 27, 32, 64 noded rectangular brick elements in three dimensional models are available for meshing.

### Loads and Boundary Conditions:

- Element sides can be given prescribed boundary conditions such as displacements, forces, pressure, inertial loads, initial displacements, initial velocities, velocity, acceleration, distributed loads, and contact loads.

### 2.4.5 Assumptions Made in the Analysis

The following assumptions are made in the present study:

1. The soil is linearly elastic and isotropic material.
2. The slippage between the wall and soil is not considered.
3. The building loads or any other structural loads are transferred on ground surface itself.
4. The lateral supports (struts) are assumed as spring elements.
5. The model assumes that there is no effect on the surrounding soil due to intrusion of the diaphragm or sheet pile wall.
6. All the computed deformations in the analysis are reported with reference to the initial equilibrium state.

#### 2.4.6 Method of Approach and Construction Sequence

The problem is analyzed directly by considering the final stage of excavation, where as the intermittent stages are not considered. In the present model the entire excavation is supported by sheet pile wall and the wall is supported laterally by struts. The soil is considered to be non homogeneous, where as the sheet pile wall has a homogeneous material. The model is analyzed using the linear static approach. As the result of the finite element analysis is independent of the number of excavation stages simulated, the present problem is analysed with direct approach and the sequential excavation process is not considered. Both in three dimensional and two dimensional models the sides are restrained laterally and the bottom is restrained in all directions. In the case of application of external loads like surcharge load, the load is applied only at the nodes of the model. To fix the length of model in the present study, a small parametric study has been carried out to find the effect of length of model on surface settlement and which shows that influence zone extends up to four times the depth excavation. Hence, the present study the length of model is taken as six times the depth of cut. The geometry of the excavation of a three dimensional model is shown in Fig. 2.6

### 2.5 Problems Considered

The following problems have been considered in the present study. The details of all the problems are given in Chapter 3.

Problem 1: Two dimensional plane strain analysis is carried out and the results are compared with the reported results of Mana and Clough [14] with the same type of problem. The soil was modeled as non-homogeneous Elastic material, where as the wall was modeled as homogeneous elastic material and the strut was modeled as elastic spring element. The parametric studies are extended to study effect of water table, effect of surface structural loads (surcharge loads), effect of embedded length of wall, and effect of strut spacing on the same two dimensional plane strain analysis.

Problem 2: Three dimensional analysis of deep excavation has been carried out. The material properties of the three dimensional model considered are same as that of two dimensional model and the soil and wall are modeled as twenty noded brick elements and the struts were modeled as two noded spring elements.

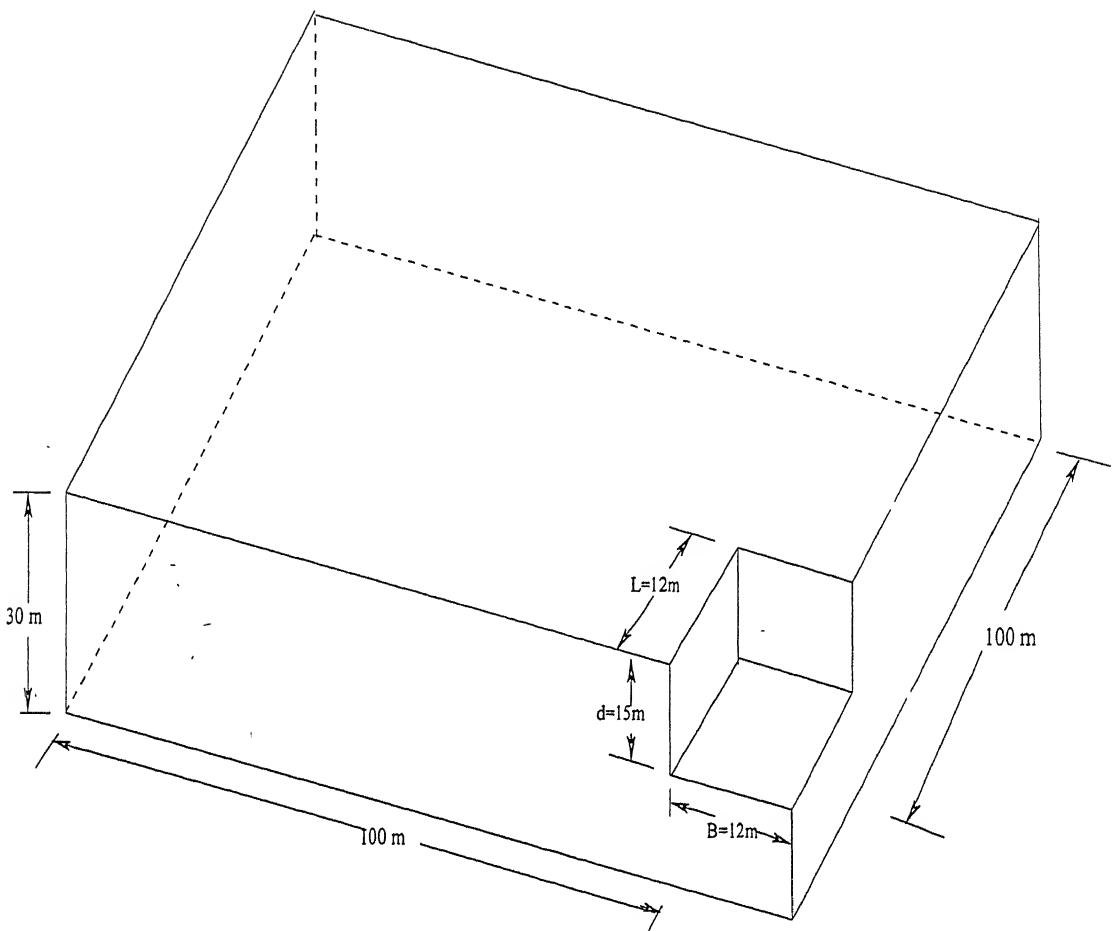


Figure 2.6: 3-D Geometry of excavation

# Chapter 3

## *Results and Discussions*

### 3.1 General

The formulation described in the previous chapter is to find the wall deformations and the ground movements around the excavations. The finite element package NASTRAN incorporating the formulation and method of solution described in the previous chapter is used for the analysis of excavation problems. The two dimensional basic configuration of the excavation model is shown in Fig. 2.1

The complete parametric studies are conducted on the two dimensional plain strain case and the results obtained are verified with existing results reported by Mana and Clough (1981) for the same data. Three dimensional analysis is carried out with the same parameters of two dimensional model with  $L/B$  ratio equal to one, where  $L$  is the length of excavation and  $B$  is the width of excavation. The details of the three dimensional geometric model are shown in Fig. 2.6. The results obtained from the three dimensional analysis are compared with the two dimensional plane strain results for the feasibility to adopt the two dimensional numerical plane strain techniques to analyse the deep excavation problems. In both the cases i.e., two dimensional and three dimensional analyses, the non homogeneous linear elastic properties of the soil and the homogeneous linear elastic properties of the wall material has been considered. The struts are considered as elastic spring elements. In this study, the influence of many parameters, viz., soil modulus multiplier, wall stiffness, strut stiffness, strut spacing in vertical direction, excavation width, depth of firm layer,  $L/B$  ratio of the excavation, water table effect, and structural loads on the ground surfaces (surcharge loads) on ground settlements and the wall movements have been analysed. The details of all the above studied parameters are given in the Table. 3.1. The results of the comparisons are presented in the form of nondimensional plots. The normalised parameters used

for the presentation of the results are summarised in the next section.

### 3.2 Effect of Specific Parameters

The results are presented using the finite element analysis performed to quantify the effects of important design parameters on maximum wall movements (MWM) and maximum surface settlements (MSS). For each case, the movement is expressed as a ratio of that which occurs using the basic parameter values listed in Table. 3.1. The ratios are assigned symbols (normalised parameters) which reflect the effect of the parameter against which is plotted. The complete normalised parameters are given below.

- 1  $\alpha_{wm}$  Ratio of MWM to MWM for a value of soil modulus multiplier 300.
- 2  $\alpha_{sm}$  Ratio of MSS to MSS for a value of soil modulus multiplier 300.
- 3  $S/h\gamma$  Normalised strut stiffness, where  $S$  is equivalent strut stiffness equal to  $AE/L$
- 4  $EI/h^4\gamma$  Normalised wall stiffness.
- 5  $\alpha_{ws}$  Ratio of MWM to MWM for  $S/h\gamma = 280$ .
- 6  $\alpha_{ss}$  Ratio of MSS to MSS for  $S/h\gamma = 280$
- 7  $\alpha_{ww}$  Ratio of MWM to MWM for a value of  $EI/h^4\gamma = 27$
- 8  $\alpha_{sw}$  Ratio of MSS to MSS for a value of  $EI/h^4\gamma = 27$
- 9  $\alpha_{wd}$  Ratio of MWM to MWM for a value of  $D=2H$
- 10  $\alpha_{sd}$  Ratio of MSS to MSS for a value of  $D=2H$
- 11  $\alpha_{we}$  Ratio of MWM to MWM for a value of  $B=0.8*H$
- 12  $\alpha_{se}$  Ratio of MSS to MSS for a value of  $B=0.8*H$
- 13  $\alpha_{wsl}$  Ratio of MWM to MWM for a value of surcharge loading  $q=0.0$
- 14  $\alpha_{ssl}$  Ratio of MSS to MSS for a value of surcharge loading  $q=0.0$
- 15  $\alpha_{wp}$  Ratio of MWM to MWM for a value of 5m spacing of struts.
- 16  $\alpha_{sp}$  Ratio of MSS to MSS for a value of 5m spacing of struts.
- 17  $\alpha_{wwt}$  Ratio of MWM to MWM for a value of water table on the ground surface.
- 18  $\alpha_{swt}$  Ratio of MSS to MSS for a value of water table on the ground surface.

### 3.3 Relationship between Maximum Wall Movements and Maximum Surface Settlements

From the available case studies on the field data, it has been found that the maximum ground settlements behind the wall vary from 0.5 to 1 times to equal to the

lateral wall movements. In the finite element case study results, the settlement ranges from 0.4-0.8 times the maximum lateral wall movements. All of these case studies are from two dimensional models only. The value of settlement obtained from the present two dimensional analysis study is 0.33 times the maximum lateral wall movement and from the three dimensional analysis study, it is 0.22 times the maximum lateral wall movement. The value obtained from the two dimensional model is nearer to the reported values from the finite element analysis and the value obtained from the three dimensional model is lower as compared to the two dimensional model.

### 3.4 Surface Settlement and Deflected Wall Profiles

To this point, the primary emphasis has been on maximum values of movement. In many cases, the shape of the deflected wall and the curvature and surface settlement are important factors. These can be quantified, using the results of the finite element analysis. The wall movement and surface settlement profiles corresponding to the different modulus multiplier values ( $M$ ) are presented in Figs. 3.1, 3.2. The profiles obtained from the present analysis matched very well with the available results from several case studies.

### 3.5 Validation of 2-D Analysis

The results obtained from the present study are compared with the results reported by Mana and Clough (1981). The details of the material properties and their variations used in the analysis are given in Table. 3.1.

The following parametric studies are carried out in the present study :

- Soil modulus
- Strut stiffness
- Wall stiffness
- Depth of firm layer
- Excavation width
- Surcharge loading
- Strut spacing
- Water table variation

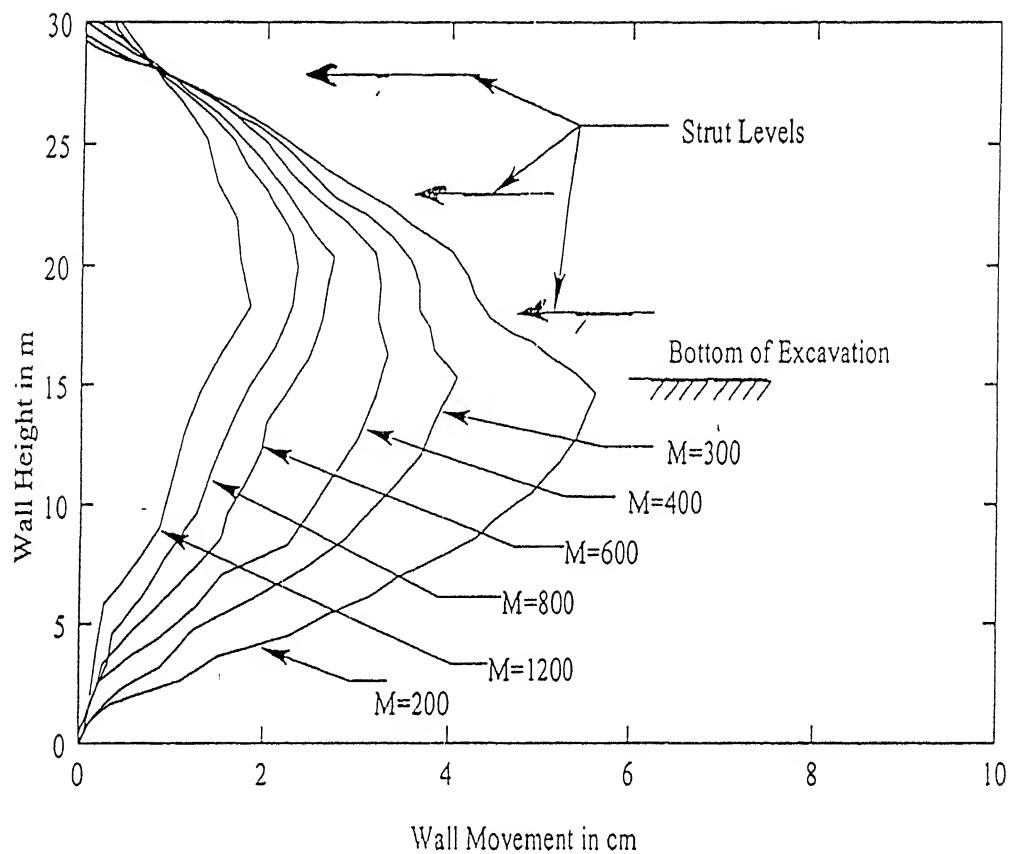


Figure 3.1: Wall Movement Profiles with Various Modulus Multiplier Values (M)

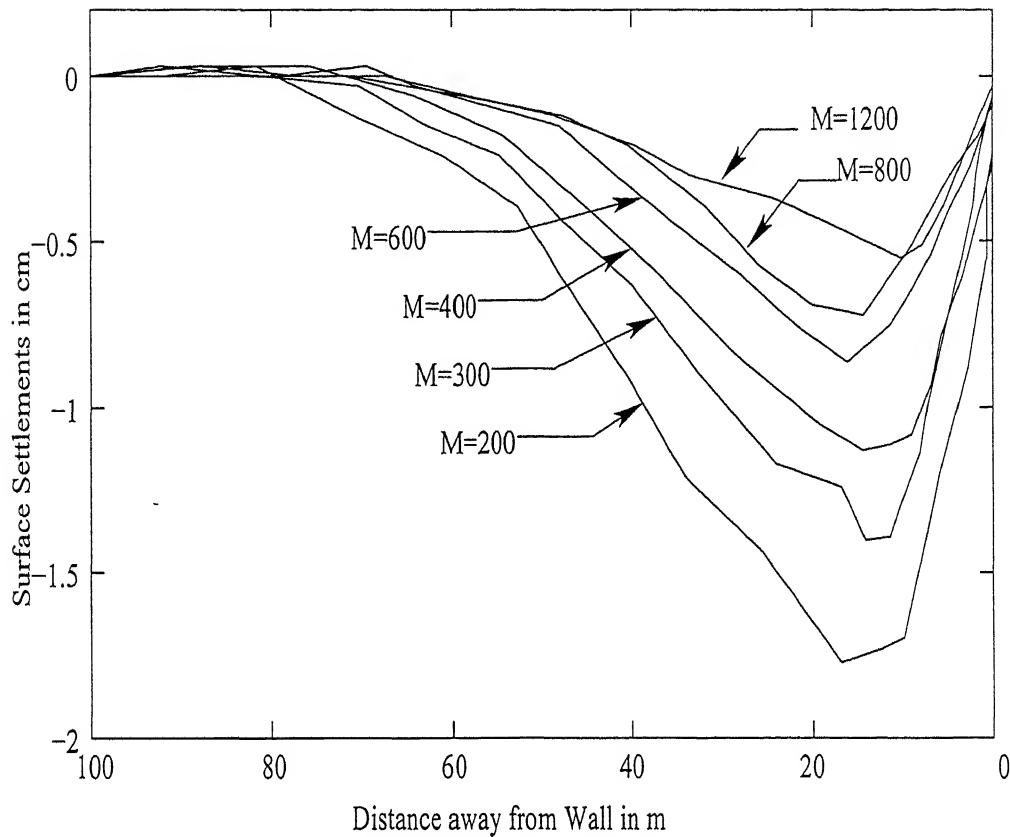


Figure 3.2: Surface Settlement Profiles for Various Modulus Multiplier Values (M)

### 3.5.1 Effect of Soil Modulus

Movement levels are strongly influenced by the soil modulus, as characterized by the modulus multiplier (M). To study the effect of soil modulus, the maximum wall movement and surface settlements are nondimensionalized with maximum wall movements and maximum surface settlements for a given value of soil modulus  $M=300$ . Higher modulus values lead to smaller movements and vice-versa for lower modulus values. Fig. 3.3 shows the variation of maximum nondimensionalized wall movements with soil modulus multiplier. The values obtained in the present study are shown in comparison with those of Mana and Clough (1981) and the results are quite similar.

The Fig. 3.4 shows the plot between the maximum surface settlement and soil modulus. The results follow the trends reported by Mana and Clough (1981).

Parameter (1)	Parameter (2)	Variations (3)
(a) Soil Conditions		
Unit weight of Soil $\gamma$ in $KN/m^3$	20	—
Undrained shear strength $S_u$ in $KN/m^2$	$28.4 + 0.2 \sigma'_v$	—
Elastic modulus of soil in $KN/m^2$	$M \times S_u$ (M=300)	M=200,400,600,800,1200
Poissons's ratio of soil $\nu$	0.48	—
(b) Support Conditions		
Wall thickness in $m$	0.268	0.167, 0.52, 0.75
Wall stiffness $EI$ in $KN - m^2/m$	$0.33 \times 10^6$	$0.125 \times 10^6, 2.44 \times 10^6, 7.35 \times 10^6$
Effective strut stiffness $AE/L$ in $KN/m/m$	$1.96 \times 10^4$	$2.8 \times 10^4, 11.21 \times 10^4, 35.03 \times 10^4$
Number of strut levels	3	2, 4, 5
Vertical strut spacing in $m$	5	8, 3.5, 2.5
Position of top strut from the top of excavation in $m$	2.0	—
(c) General Conditions		
Depth of firm layer in $m$	30	15, 21, 25
Final depth of excavation in $m$	15	—
Width of excavation in $m$	12	15, 22, 30, 48
Surcharge loading in $KN/m^2$	0	75, 150, 30

Table 3.1: Basic Conditions and Variations for Parametric Study of Structed Excavations

### 3.5.2 Effect of Strut Stiffness

The effect of strut stiffness has been studied on maximum wall movement and maximum surface settlement. The strut stiffness is normalized as  $S/h\gamma$ , where  $h$  is the spacing of struts in vertical direction and  $S$  is the effective strut stiffness. The normalized strut stiffness is plotted on the logarithmic scale. The deflections are normalized with the deflections for a normalized strut stiffness of 280.

The variation of normalized wall movement with strut stiffness is shown in Fig. 3.5. The trend of the results obtained is very much similar to those reported by Mana and Clough, but the values differ slightly for higher values of strut stiffness. The increase in strut stiffness causes a decrease in wall movement, but the effect shows the diminishing returns at very high values of strut stiffness.

In Fig. 3.6. the variation of maximum surface settlement is shown in comparison with the results of Mana and Clough. The results matched well with the reported values.

### 3.5.3 Effect of Wall Stiffness

To study the effect of wall stiffness, the wall stiffness is normalized as  $EI/h^4\gamma$ . The maximum wall movement and maximum surface settlement are normalized with the deflection values for a normalized wall stiffness of 27. The normalized wall stiffness is plotted on the logarithmic scale.

The variation of the maximum wall movement with wall stiffness is shown in Fig. 3.7. The maximum wall movements are decreasing with increase in the stiffness of wall. The results are compared with the results obtained by Mana and Clough. They are similar in trend except for a slight difference in values.

The variation of maximum surface settlement with wall stiffness is shown in Fig. 3.8. The results almost follow the same trend of the results reported by Mana and Clough. The values are also similar with the reported data. The surface settlements decrease with increase in wall stiffness.

### 3.5.4 Effect of Depth of Firm Layer

The effect of depth of firm layer on maximum wall movement and the maximum surface settlement have been studied. The normalized deflections are plotted with the depth of firm layer (D). The conventions for the depth of firm layer are shown in Fig. 3.9. The depth of firm layer is expressed in terms of depth of excavation (H).

The variation of the maximum wall movement with the depth of firm layer is shown in Fig. 3.9 and the results of the present study are compared with the reported results of Mana and Clough. There is good agreement between the present data with the reported results.

In Fig. 3.10, the variation of maximum surface settlements to the depth of firm layer is plotted and the present results are compared with the reported results. The trend of the present data and the reported data are same, except for slight variation in their values. The main observation is that the movements increased in both the cases

as the depth of firm layer is increased.

### 3.5.5 Effect of Excavation Width

The effect of excavation width on maximum wall movement and maximum surface settlement has been studied. To study this effect the complete model has considered for the analysis and the deflections are normalized with the basic conditions. The normalized deflections are plotted with the excavation width, which is expressed in terms of excavation depth (H).

The variations of normalized wall movements with the excavation width are plotted in Fig. 3.11. The results obtained from the present study are compared with the results reported by Mana and Clough. There is a good agreement between the present results and the reported results except for a slight change in the rate of variation. In the present study, the rate of variation decreases with the excavation depth but the reported data shows that the rate is a constant.

Figure 3.12 shows the variation of normalized surface settlement with the excavation width. The present results are compared with the reported results by Mana and Clough. The rate of change in settlement is decreasing with the increase in excavation width. Initially both the data are comparable but for maximum excavation width the results are differing. The deflections are increased with the increase in excavation width.

### 3.5.6 Effect of Surcharge Load

The effect of surcharge load is studied on both the maximum wall movement and the maximum surface settlement. In Fig. 3.13 the normalized deflections are plotted with surcharge loading and it can be noted that both the wall movements and the surface settlements are linearly increasing with increase in surcharge loading.

### 3.5.7 Effect of Strut Spacing

The effect of strut spacing on the variation of maximum wall movement and maximum surface settlements have been presented in Fig. 3.14. The deflections are normalized with 5 meter vertical spacing of struts. Both the movement of the wall and the surface settlement are continuously increasing with the increase in vertical spacing of struts.

### 3.5.8 Effect of Water Table

The effect of depth of water table has been studied on the variation of maximum wall movements and maximum surface settlements. The normalized deflections are plotted

with the water table at top of the surface. The deflections are decreasing with increase in depth of water table. The main observation found from the data was that the rate of change of deflections are initially very high but the rate is almost constant later on. Figure 3.15. shows the normalized deflections with the effect of water table variation.

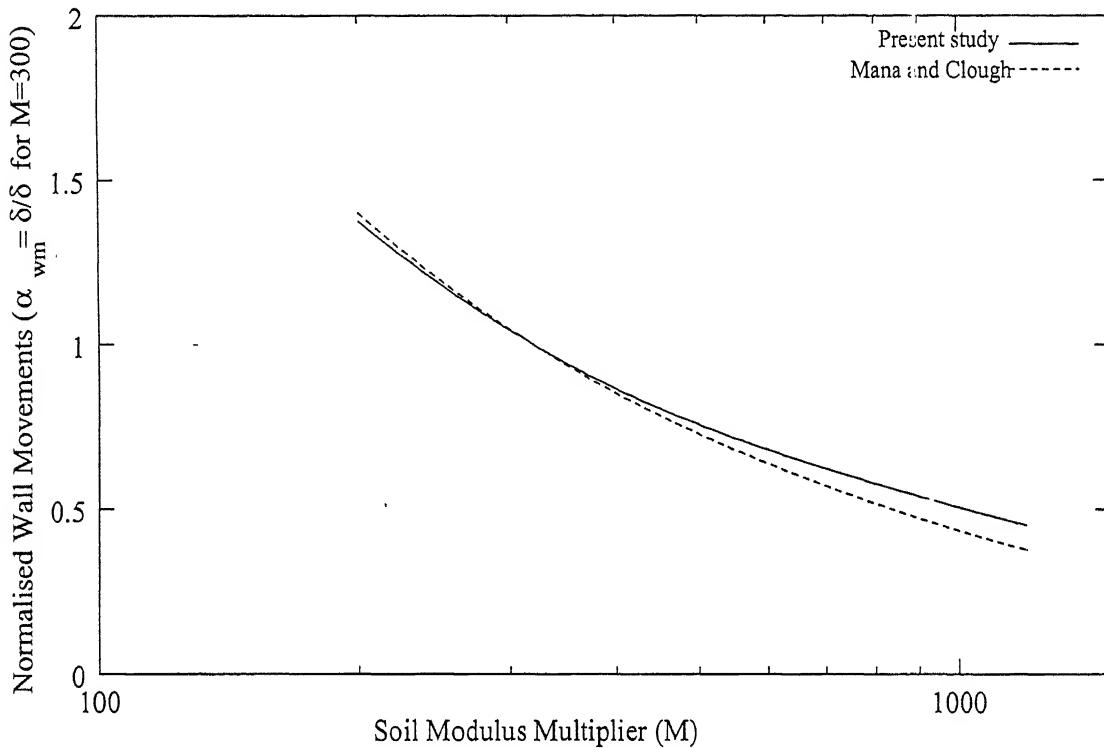


Figure 3.3: Effect of Soil Modulus Multiplier(M) on Maximum Wall Movement

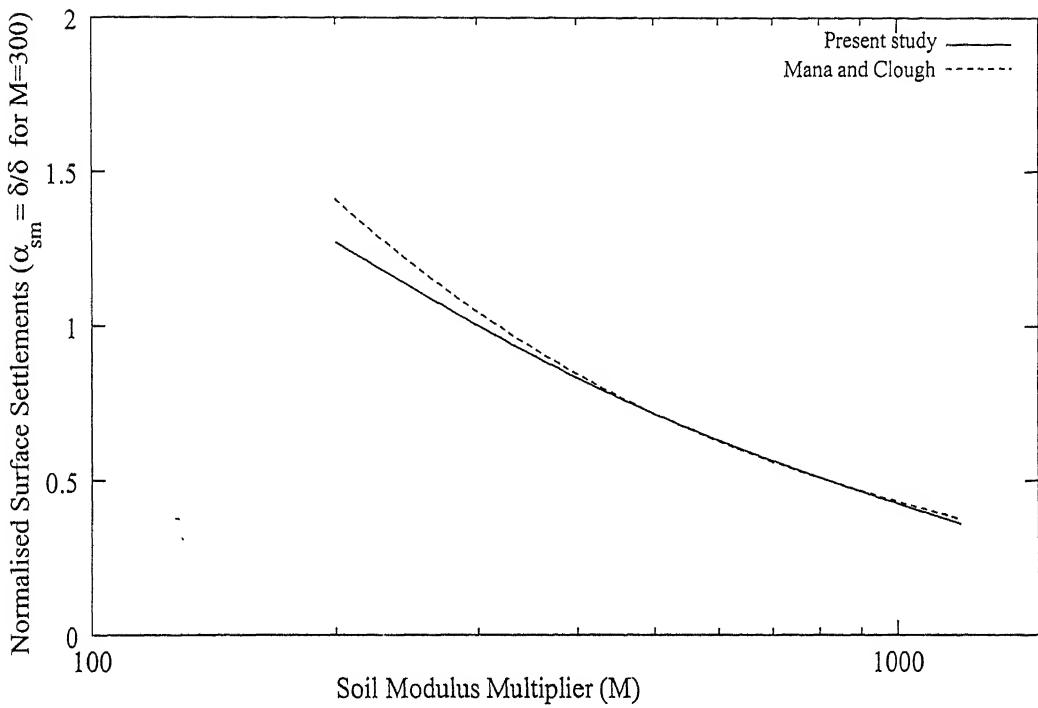


Figure 3.4: Effect of Soil Modulus Multiplier( $M$ )on Maximum Surface Settlement

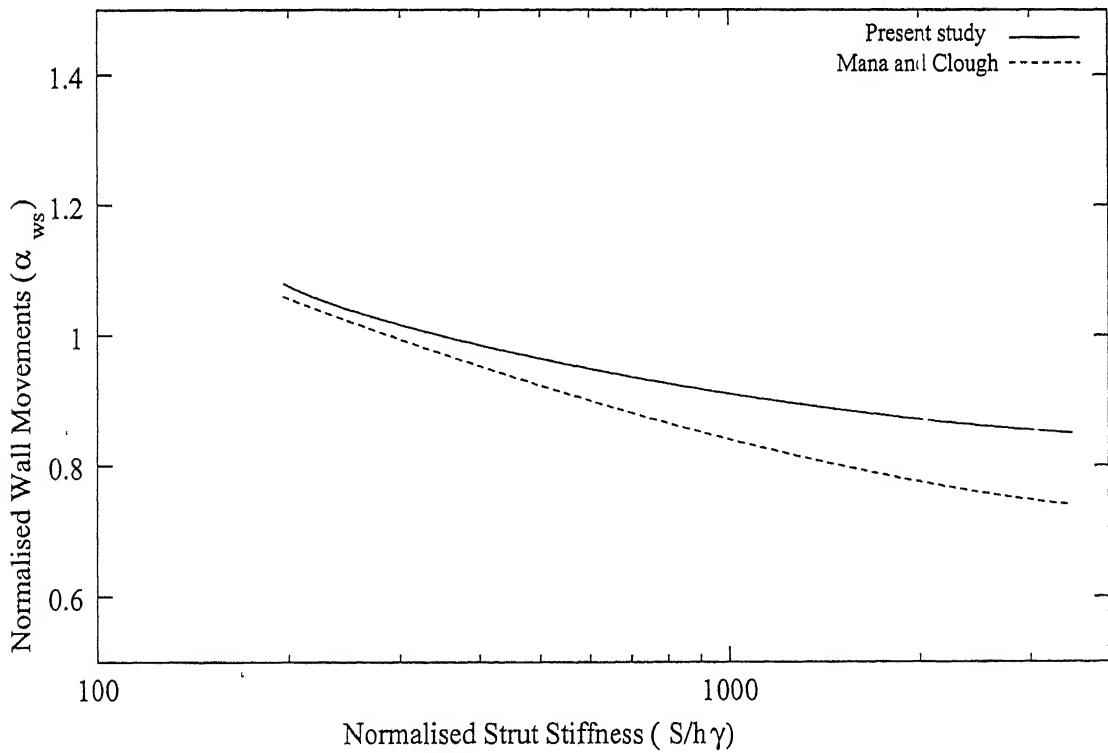


Figure 3.5: Effect of Strut Stiffness on Maximum Wall Movement

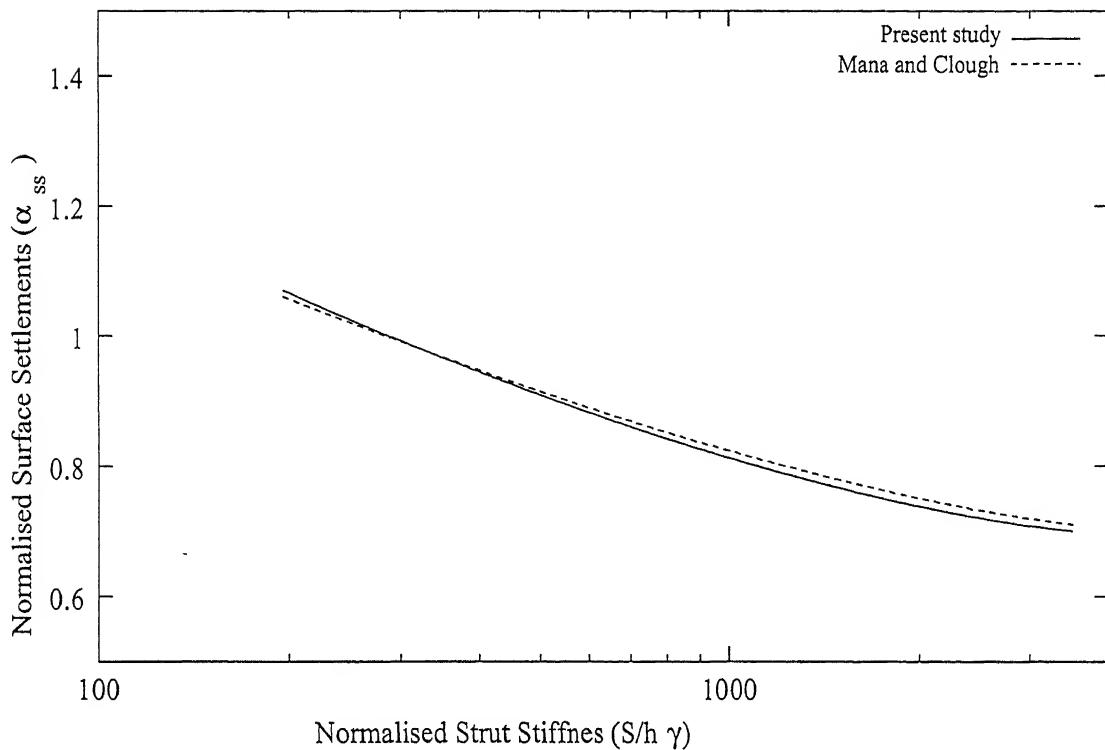


Figure 3.6: Effect of Strut Stiffness on Maximum Surface Settlement

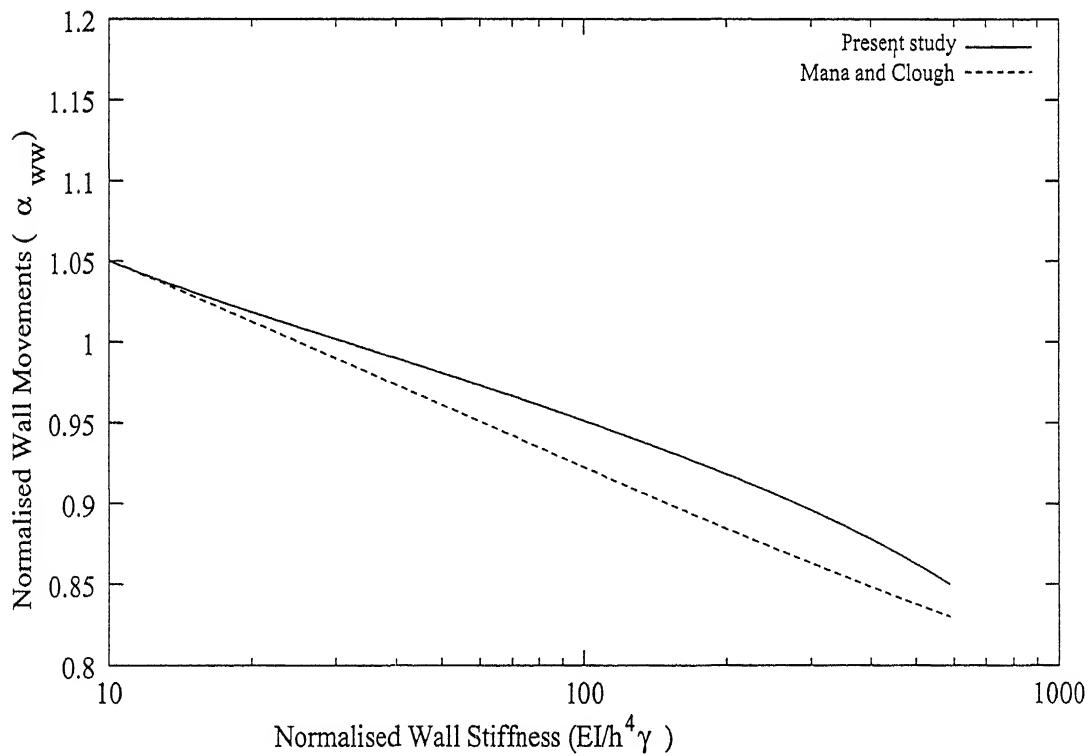


Figure 3.7: Effect of Wall Stiffness on Maximum Wall Movement

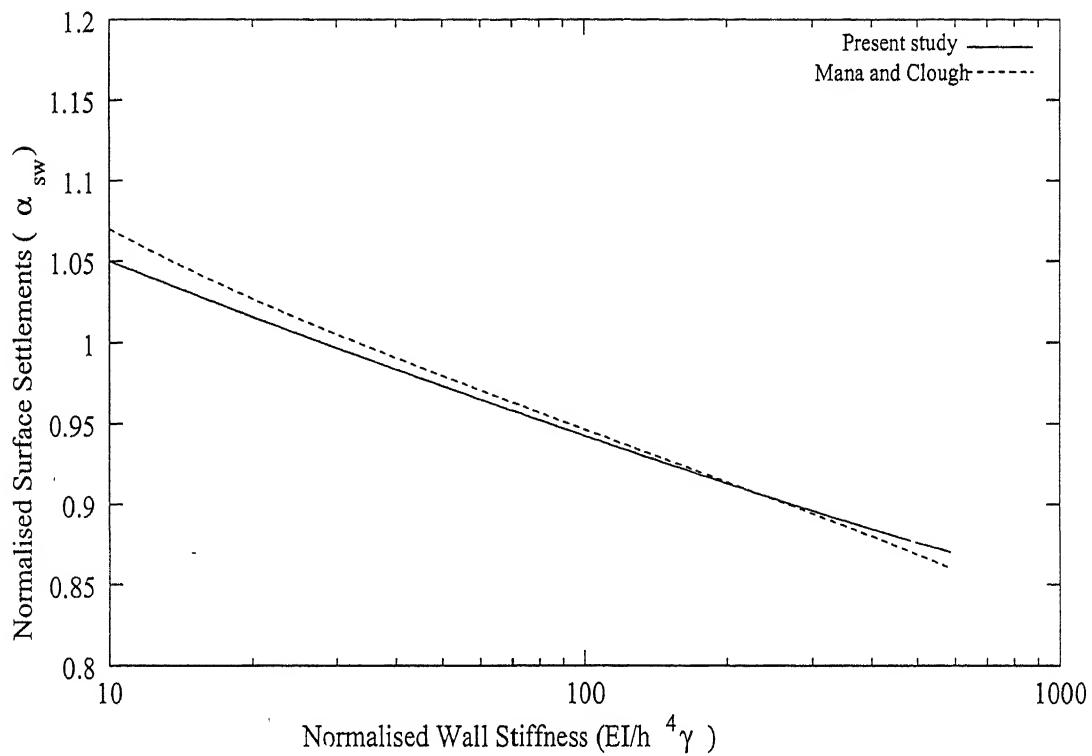


Figure 3.8: Effect of Wall Stiffness on Maximum Surface Settlement

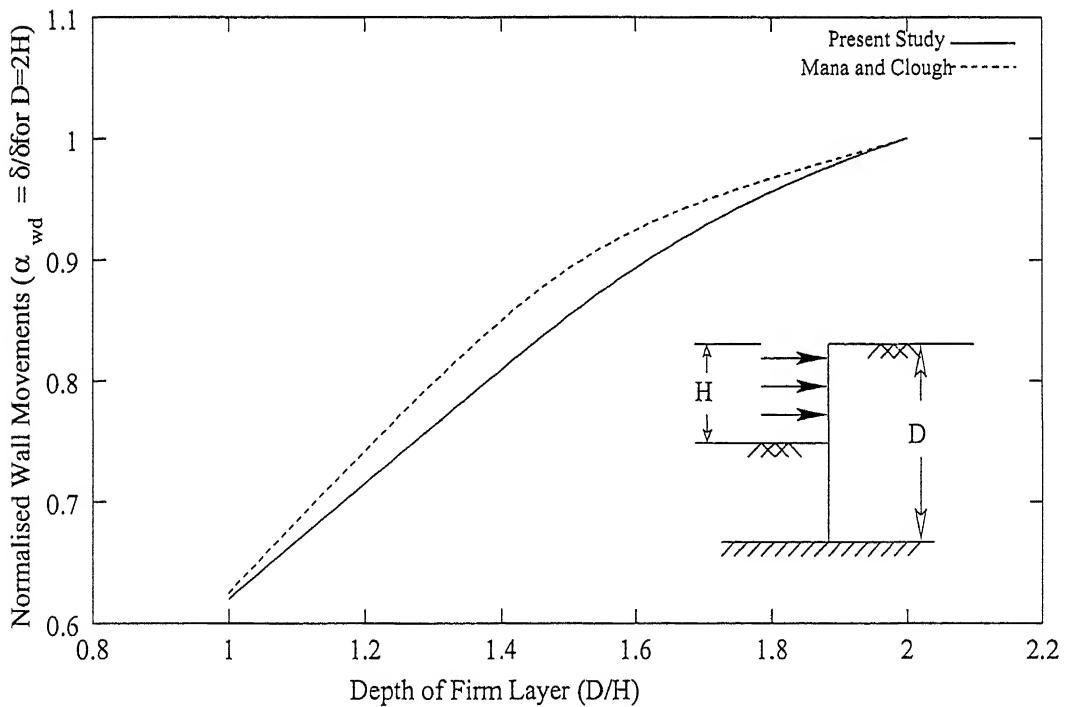


Figure 3.9: Effect of Depth to Underlying Firm Layer on Maximum Wall Movement

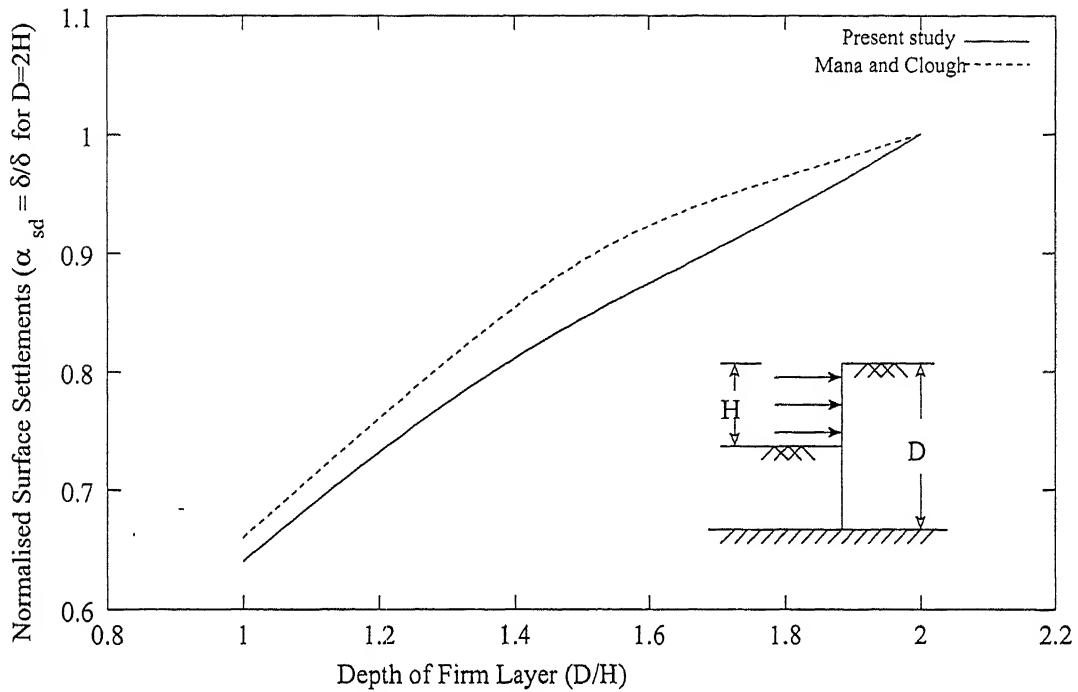


Figure 3.10: Effect of Depth to Underlying Firm Layer on Maximum Surface Settlement

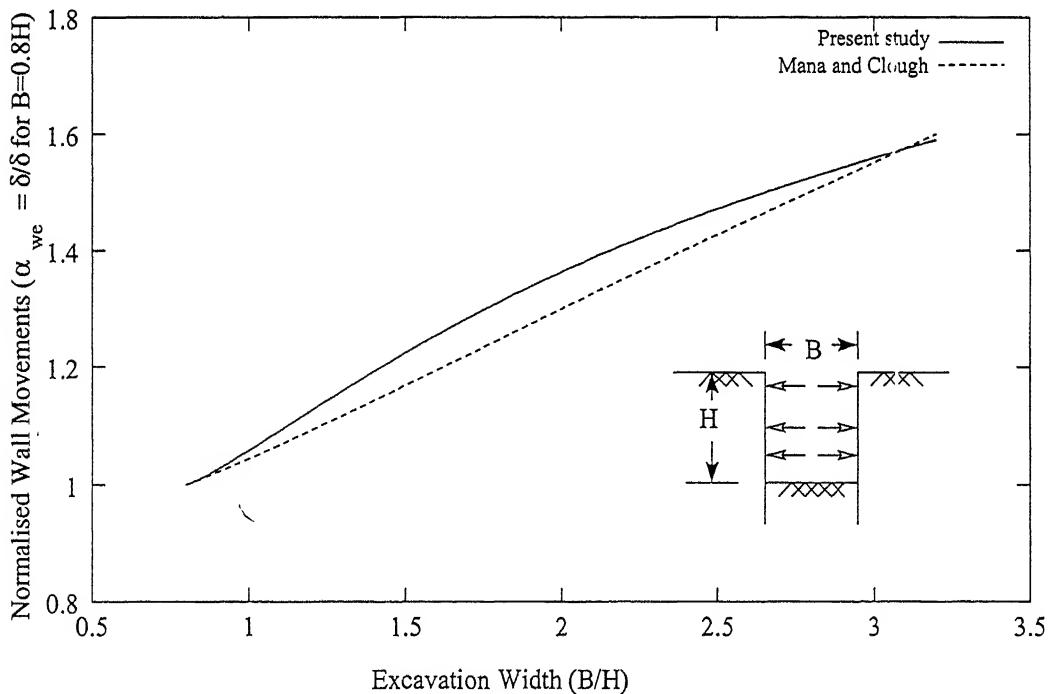


Figure 3.11: Effect of Excavation Width on Maximum Wall Movement

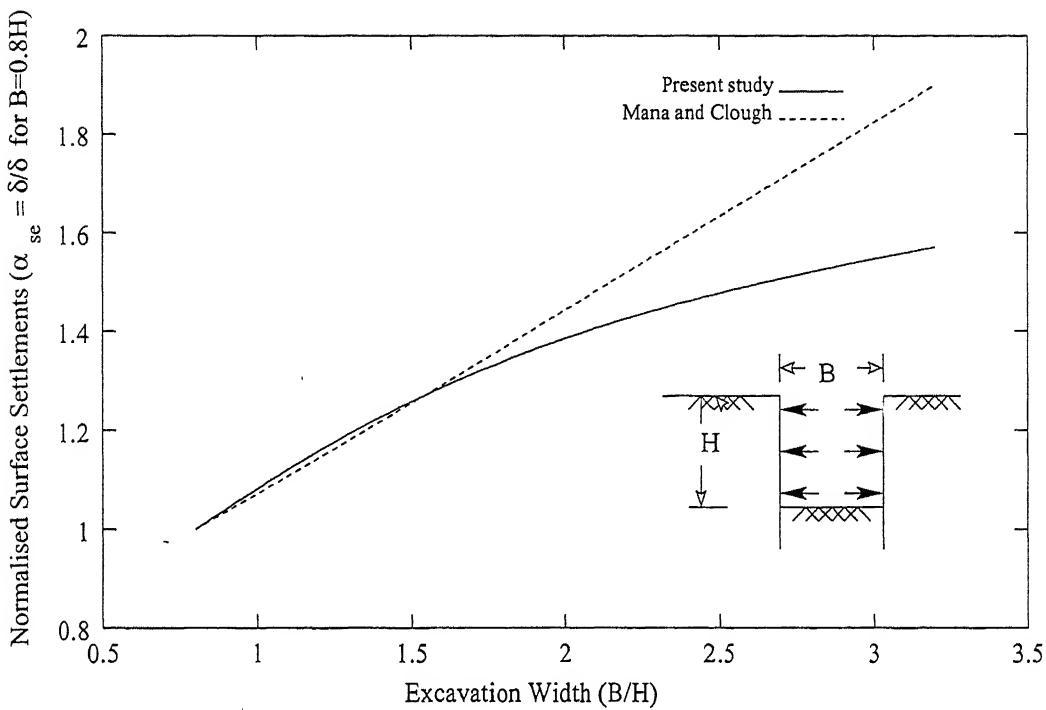


Figure 3.12: Effect of Excavation Width on Maximum Surface Settlement

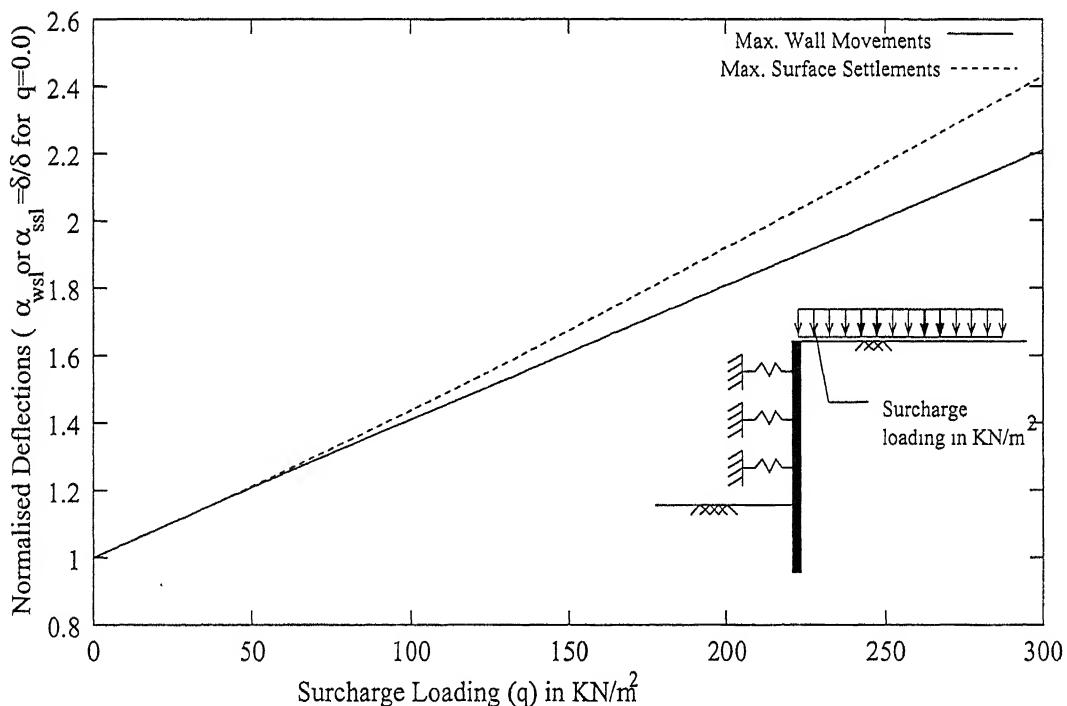


Figure 3.13: Effect of Surcharge Loads on Maximum Wall Movements and Surface Settlements

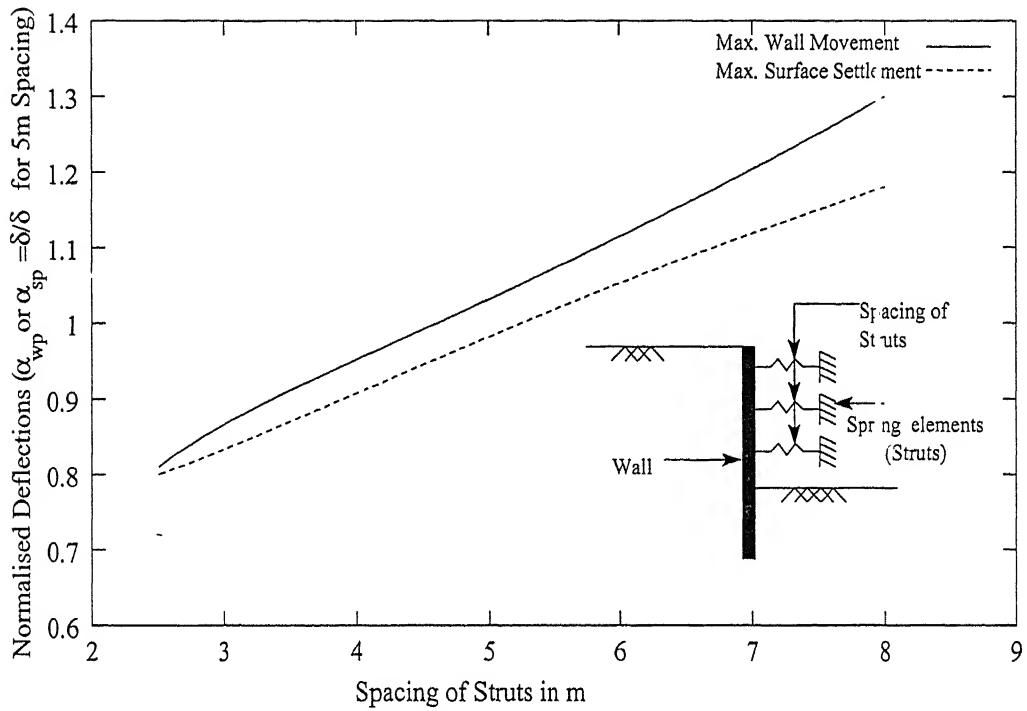


Figure 3.14: Effect of Spacing of Struts on Maximum Wall Movement and Surface Settlement

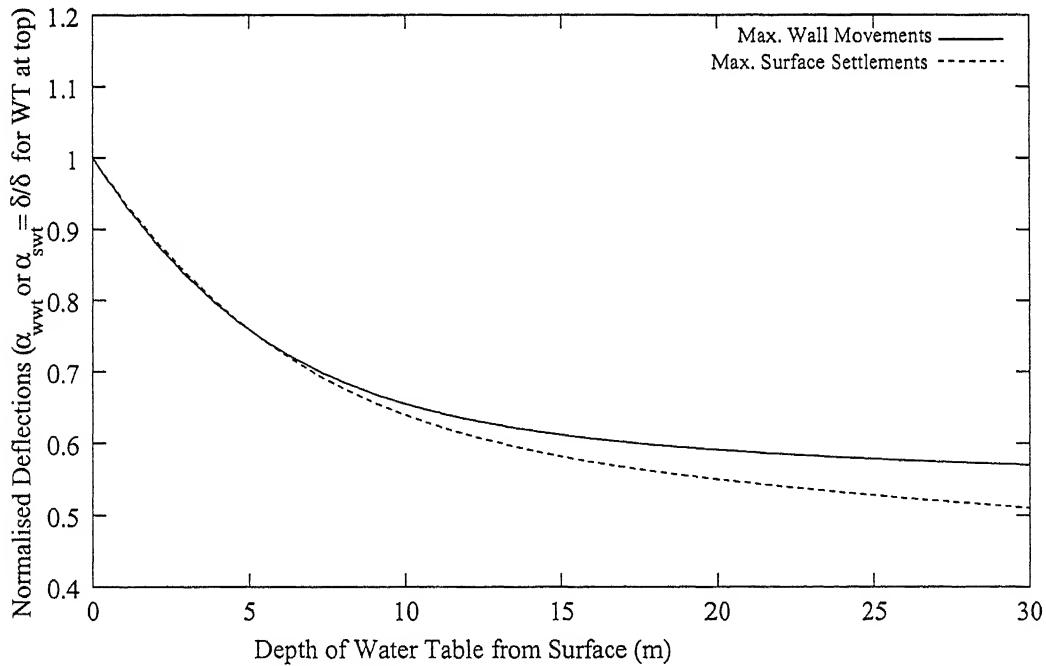


Figure 3.15: Effect of Water Table on Maximum Wall Movement and Surface Settlement

### 3.6 3-D Analysis and Validation of 2-D plane strain analysis

Three dimensional analysis of struttured deep excavation has been carried out, and the geometric model of the excavation is shown in Fig. 2.6 with  $L/B=1$ . The results obtained from the analysis are presented in the form of nondimensional plots. For the purpose of comparison the absolute values of maximum wall movements and maximum surface settlements from two dimensional and three dimensional analysis with variation in soil Elastic modulus are given in Table. 3.2. In the three dimensional analysis the soil conditions, support conditions and the basic conditions are considered to be same as those of two dimensional case. The only extra parameter needed in the three dimensional case is horizontal spacing of struts and in the present case it is taken as 2 meter c/c. The maximum wall movements and the maximum surface settlements are considered at the section of symmetry i.e., at the center of excavation.

Modulus Multiplier	2-D Model results		3-D Model results	
	Max. Wall Movement in cm	Max. Surface Settlement in cm	Max. Wall Movement in cm	Max. Surface Settlement in cm
200	5.61	1.76	4.8	1.08
300	4.08	1.39	3.9	0.81
400	3.315	1.10	3.31	0.58
600	2.73	0.86	2.56	0.44
800	2.325	0.695	2.10	0.39
1200	1.83	0.49	1.55	0.264

Table 3.2: Comparison of Absolute Values of Displacements

The effect of the following parameters on the maximum wall movements and maximum surface settlements are studied.

- Soil modulus
- Strut stiffness
- Wall stiffness
- Length to width ratio of excavation
- Surcharge load on ground surface
- Spacing of struts

### 3.6.1 Effect of Soil Modulus

The effect of soil elastic modulus has been studied using three dimensional analysis. The variation in normalized wall deflections and surface settlements with soil modulus multiplier is shown in Fig. 3.16. The results obtained from the three dimensional analysis are compared with the two dimensional plane strain analysis. In Fig. 3.17. the trend of three dimensional analysis results are following the similar to that of two dimensional analysis. The absolute values of wall movements obtained from the three dimensional analysis are same as that of two dimensional plane strain analysis, though the absolute values of surface settlements obtained from three dimensional analysis are much less than that of two dimensional plane strain values. But the values obtained from the two dimensional plane strain analysis are very well agreeable with the three dimensional analysis. This leads to much advantage of two dimensional plane strain analysis over three dimensional analysis with respect to computational time and memory space.

### 3.6.2 Effect of Strut Stiffness

The variation of normalized maximum wall movement and surface settlement with strut stiffness is shown in Fig. 3.18. The two dimensional and three dimensional analysis results of the normalized deflections with strut stiffness are plotted in Fig. 3.19. The maximum wall movements and surface settlements decrease with increase in strut stiffness and the trends are very much similar. But the effect shows diminishing returns at very high values of strut stiffness in both of the analyses and it is much higher in case of two dimensional analysis.

### 3.6.3 Effect of Wall Stiffness

In Fig. 3.20 the variation of normalized wall deflections and surface settlements with stiffness of wall is presented. The comparison of two dimensional and three dimensional results of varying normalized deflection with stiffness of wall is shown in Fig. 3.21. The normalized maximum wall movements and surface settlements are very high from three dimensional analysis as compared to two dimensional analysis, the rate of change in the deflections are also more in the three dimensional analysis and the rate of change is very low in two dimensional analysis. The three dimensional model is better to study the effect of wall rigidity.

### 3.6.4 Effect of Surcharge Load on Ground Surface

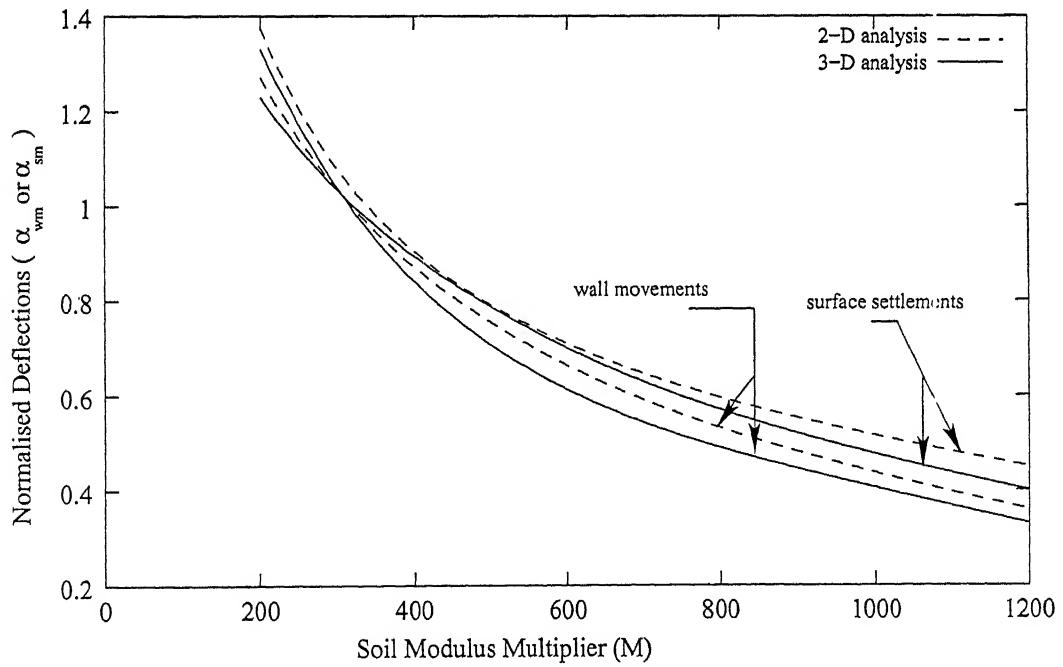
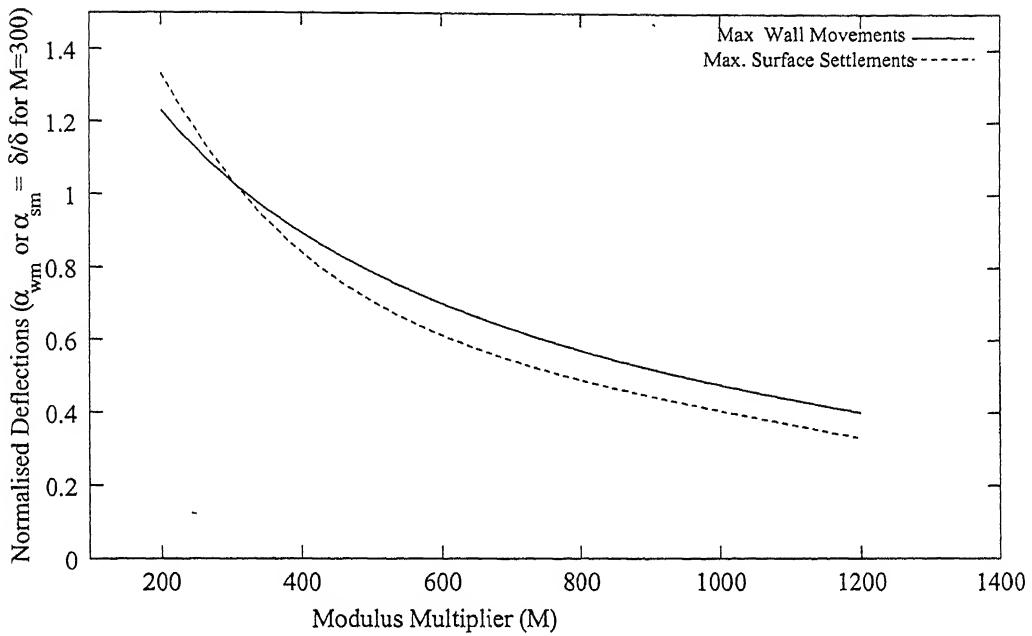
The variation of normalised wall deflections and surface settlements with the surcharge load is presented in Fig. 3.23. In Fig. 3.24. plots of two dimensional and three dimensional analysis of the maximum wall movement and the surface settlement with surcharge load has been presented. The deflections are normalised with deflections corresponding to zero surcharge load. The wall deflections and the surface settlements are linearly increasing with the increase in surcharge load. There is a good agreement between the two dimensional and three dimensional results.

### 3.6.5 Effect of Spacing of Struts

The variation of normalised wall deflections and surface settlements with the spacing of struts have been given in Fig. 3.24. The variation of the two dimensional and three dimensional analysis of the maximum wall movement and the surface settlement with spacing of struts have been given in Fig. 3.25. The deflections are normalized with the vertical spacing of struts equal to 5 meter and the horizontal spacing of struts are constant for all the cases and the value of horizontal spacing of struts is 2 meter. The wall movements and the surface settlements increase with the increase in strut spacing. The rate of change of movement and settlement are almost constant and the two dimensional wall movements are slightly higher than the three dimensional wall movements.

### 3.6.6 Effect of Length to Width Ratio of Excavation

The main problem with the two dimensional plane strain analysis is the effect of length to width ratio of excavations. The excavations are three dimensional in nature. The variations of length to width ratio of the excavation on wall movement and surface settlement are shown in Fig. 3.26. The deflections are normalized with deflections for  $L/B = 1$ . The movements of the wall and the surface settlements increase with the increase in length to width ratio of excavation. The rate of change is more for the initial  $L/B$  ratio, after certain stage of  $L/B$  ratio the rate of change is almost constant.



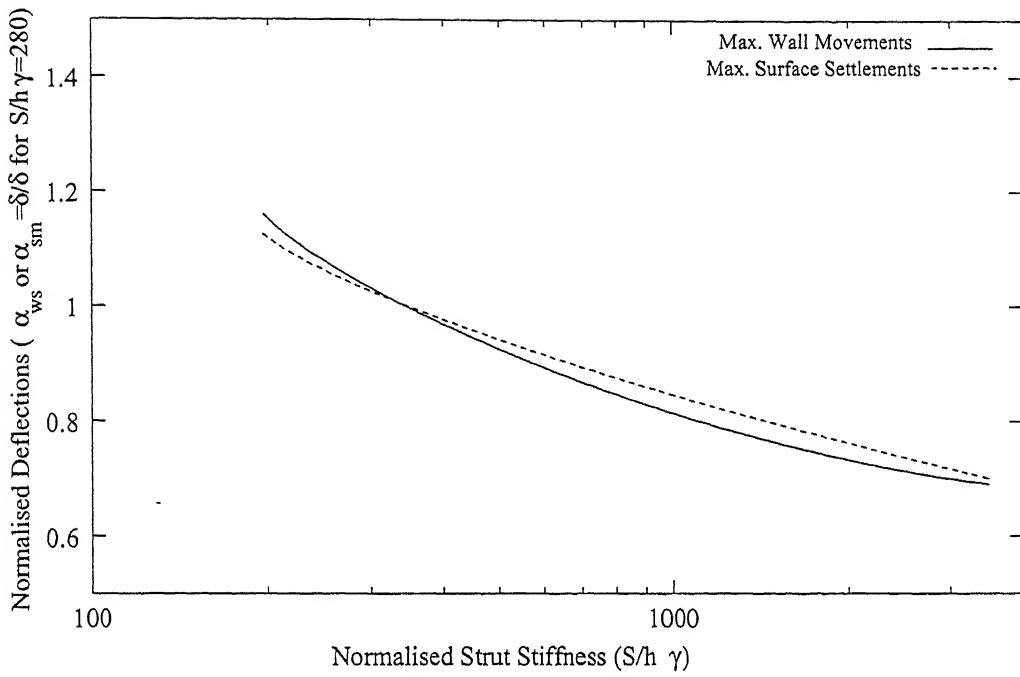


Figure 3.18: Effect of Strut Stiffness on Maximum Wall Movement and Surface Settlement

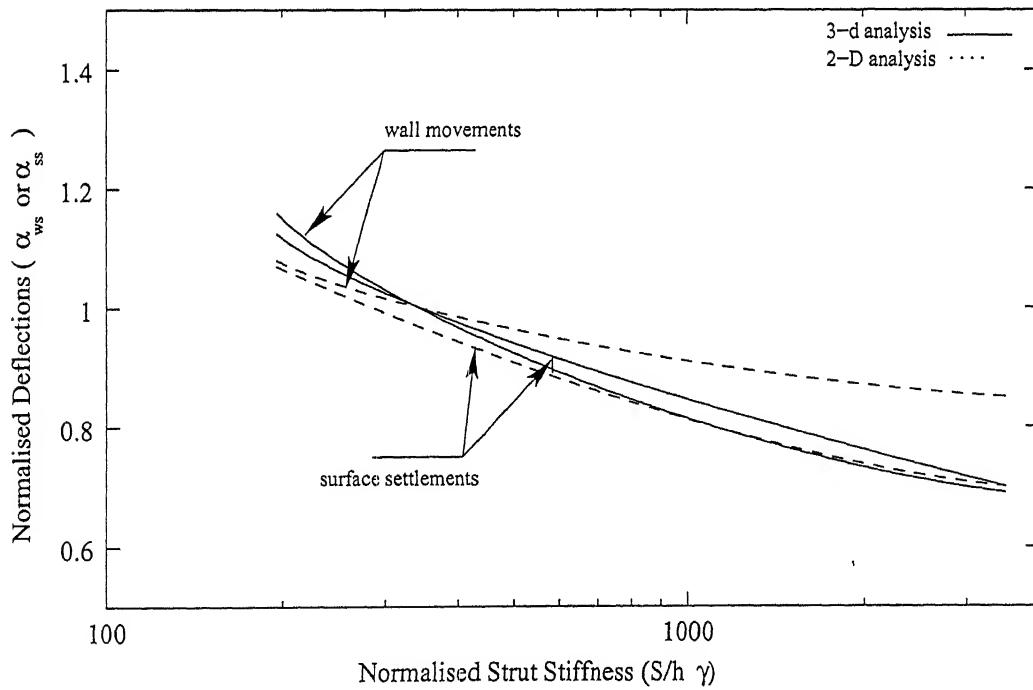


Figure 3.19: Effect of Strut Stiffness on Maximum Wall Movement and Surface Settlement

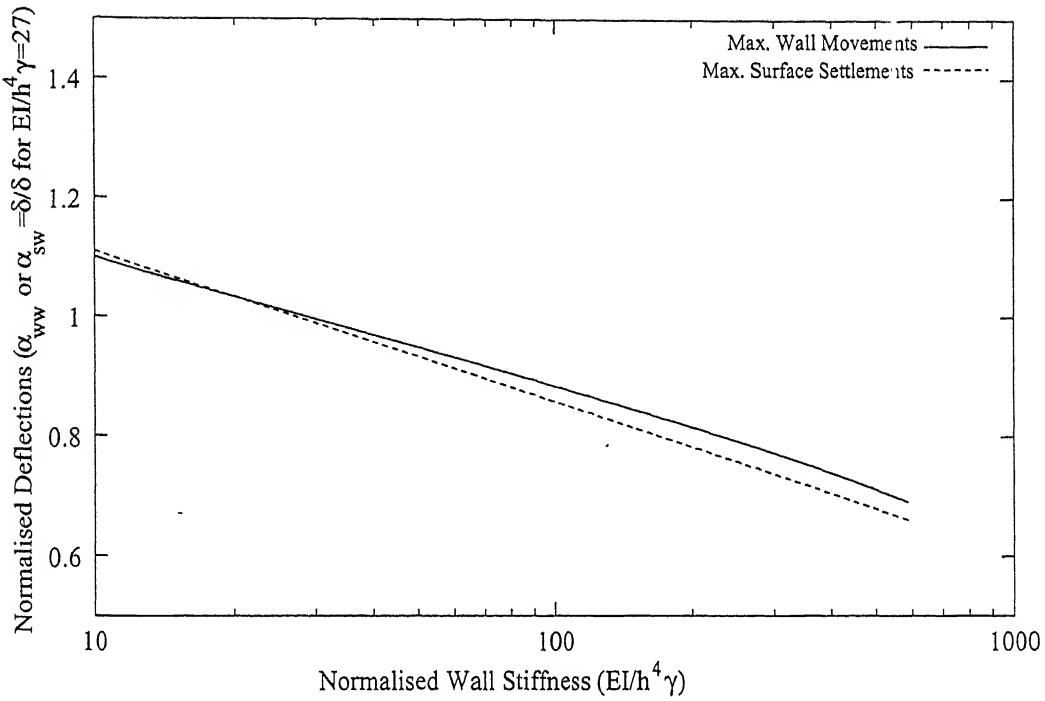


Figure 3.20: Effect of Wall Stiffness on Maximum Wall Movement and Surface Settlement

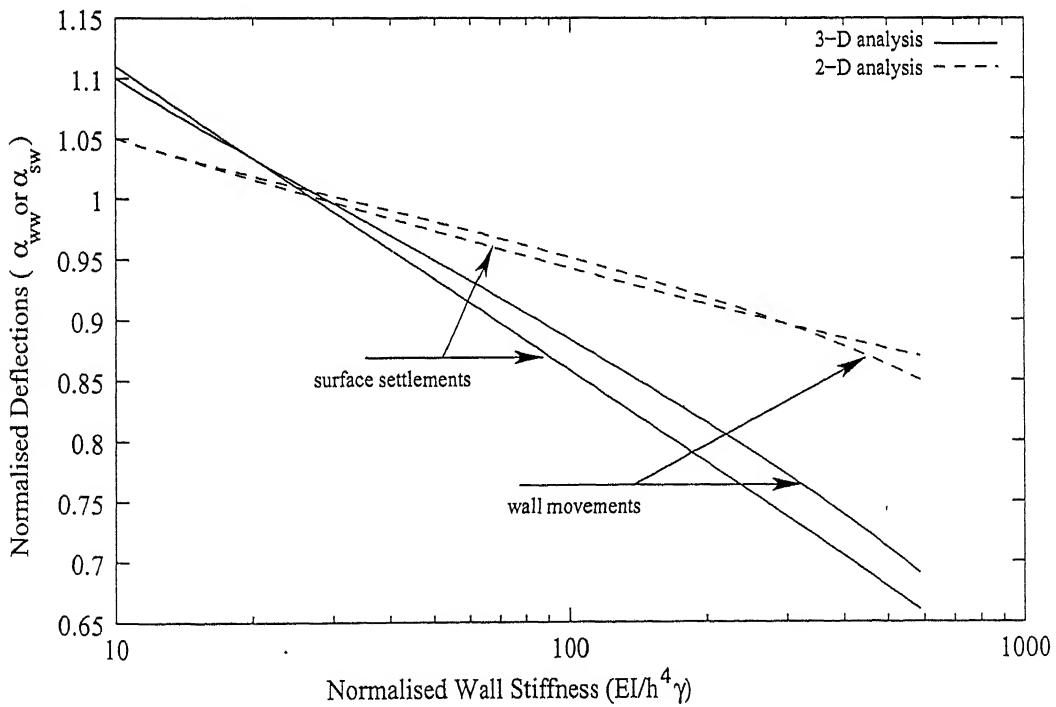


Figure 3.21: Effect of Wall Stiffness on Maximum Wall Movement and Surface Settlement

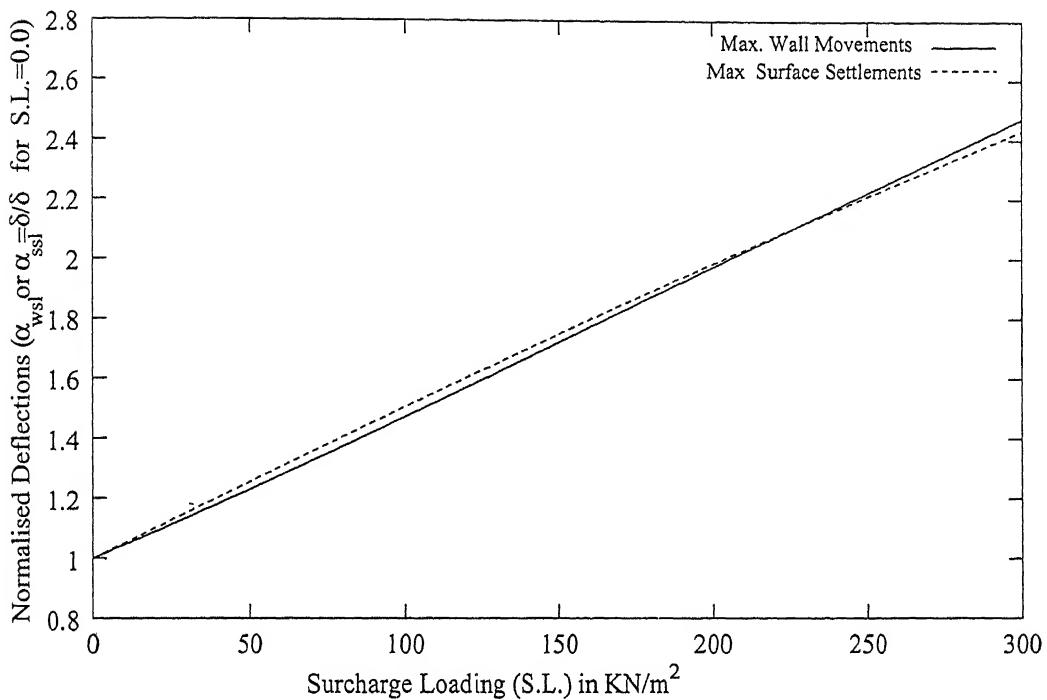


Figure 3.22: Effect of Surcharge Load on Maximum Wall Movement and Surface Settlement

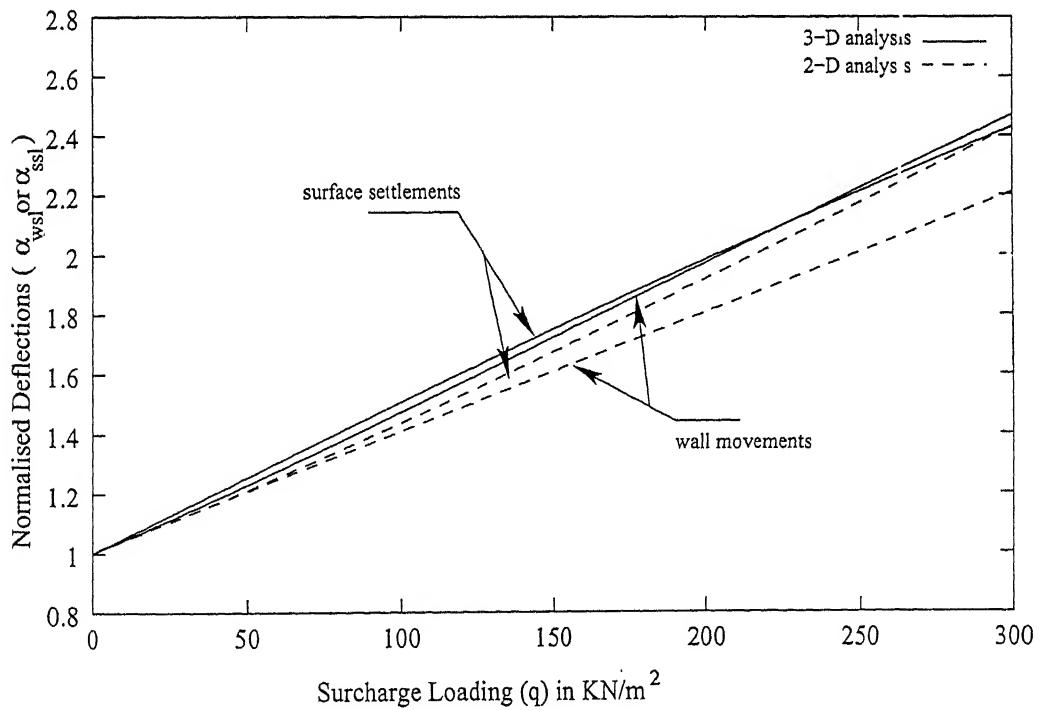


Figure 3.23: Effect of Surcharge Load on Maximum Wall Movement and Surface Settlement

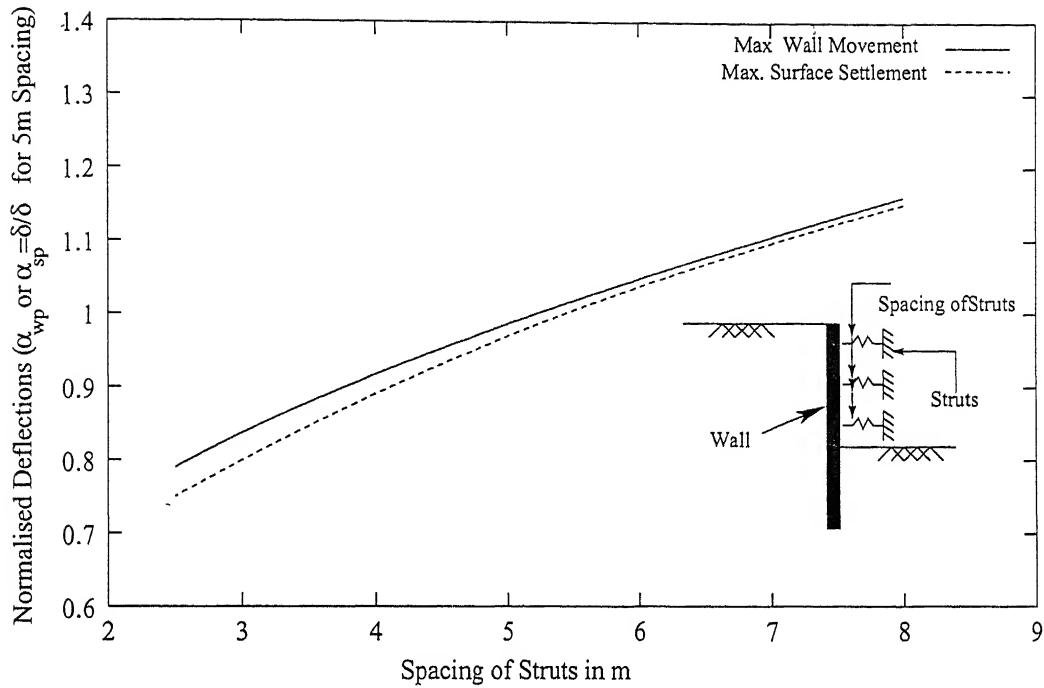


Figure 3.24: Effect of Strut Spacing on Maximum Wall Movement and Surface Settlement

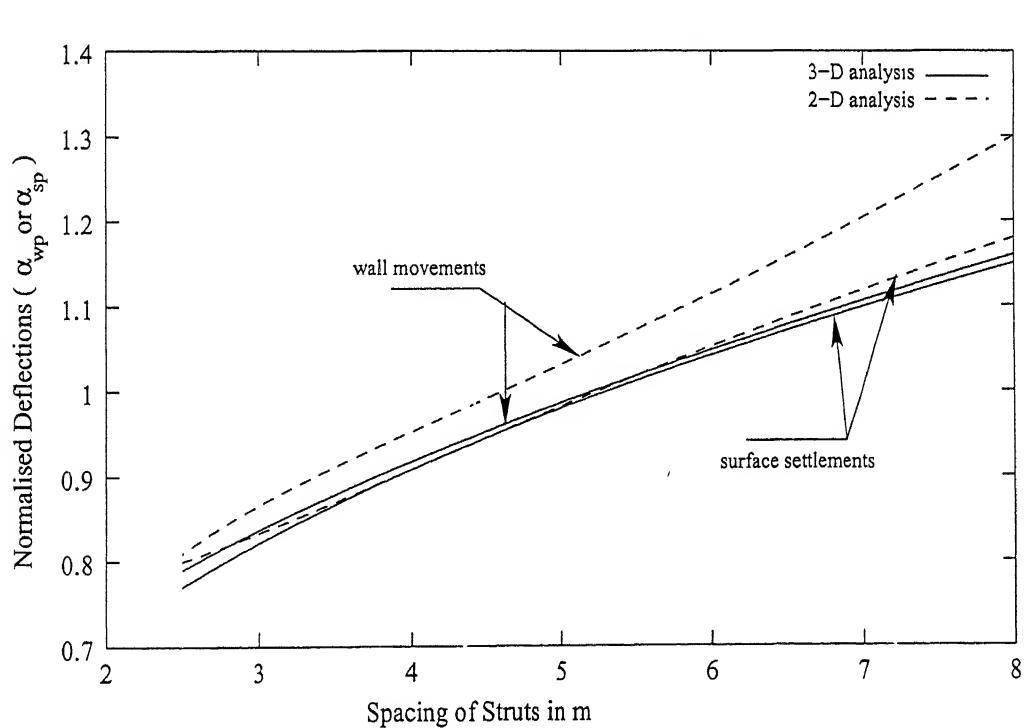


Figure 3.25: Effect of Strut Spacing on Maximum Wall Movement and Surface Settlement

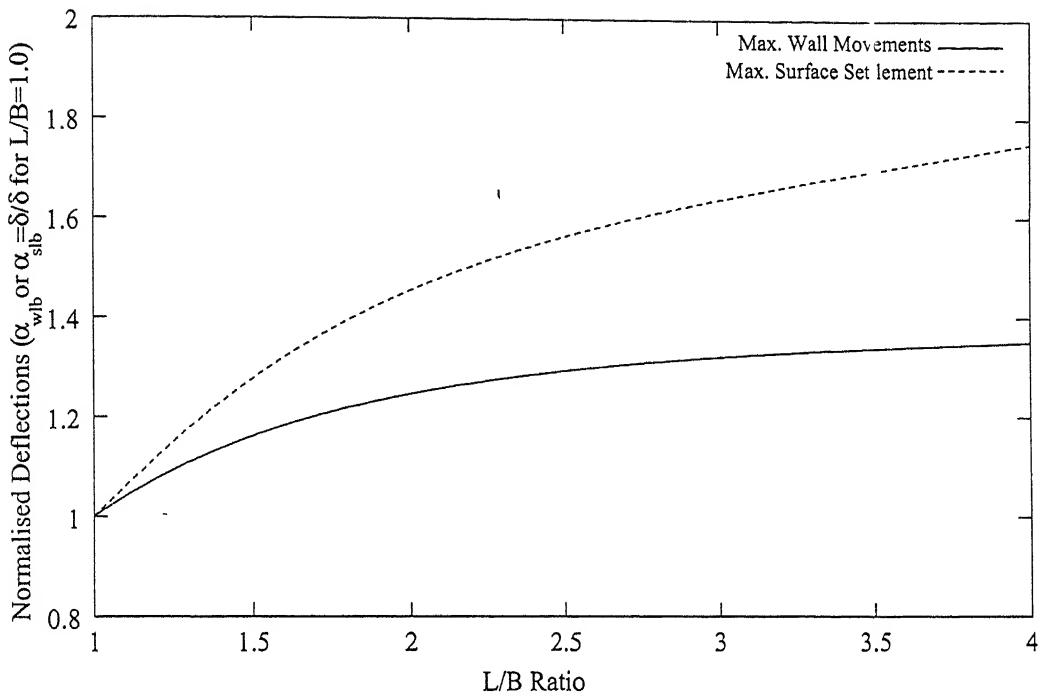


Figure 3.26: Effect of L/B Ratio on Maximum Wall Movement and Surface Settlement

# Chapter 4

## *Conclusions and Scope for Future Study*

### 4.1 General

The results obtained in the present study for different parametric studies have already been discussed in the previous chapter. The conclusions, possible extensions of the study and scope are presented in the following sections.

### 4.2 Conclusions

The problem of struttured deep excavation has been studied through finite element analysis using finite element package NASTRAN. Based on the results obtained in this study the following conclusions are drawn.

1. The results obtained from two dimensional analysis are compared with the results reported in literature. The results matched well with the reported results.
2. The patterns in the variation of maximum wall movements and maximum surface settlements shown by three dimensional analysis results follow similar trends as of two dimensional analysis. The absolute values of wall movements obtained from three dimensional analysis are same as that of two dimensional plane strain analysis, though the absolute values of surface settlements obtained from three dimensional analysis are much less than that of two dimensional plane strain analysis.
3. The maximum wall movements and maximum surface settlements decrease with the increase in elastic modulus of soil and for high elastic modulus very small movements are observed.

4. Increasing strut stiffness decreases movements, but effect shows diminishing returns at very high values of strut stiffness.
5. The wall movements and surface settlements decrease with increase in wall stiffness, and the decrease in three dimensional analysis is higher as compared to two dimensional plane strain analysis.
6. The wall movements and surface settlements are increased as excavation width and depth to an underlying firm layer are increased.
7. The wall movements and surface settlements are increased directly with the increase in surface surcharge loads and the trend is same in both two dimensional and three dimensional analysis.
8. Increase in depth of water table from surface results in decrease in maximum wall movements and maximum surface settlements.
9. The maximum wall movements and maximum surface settlements increase with increase in length to width ratio of excavation and the rate of change decreases.
10. The wall movements and surface settlements decrease with decrease in spacing of struts.
11. Most of the results obtained from the two dimensional plane strain analysis compare very well with the three dimensional analysis except in the case of the effect of wall stiffness.

### **4.3 Scope for Future Study**

In the present study the effect of parameters which influence the ground movements of deep excavations have been studied, by considering soil behavior as linearly elastic material. Few possible topics which can be studied further in this area are as follows,

1. Incorporating the effect of slippage between the soil-wall interface.
2. Study of arbitrary shaped excavations.
3. Incorporating bar elements in three dimensional analysis.
4. Incorporating nonlinear material models.
5. Incorporating the solution with Mohr's coulomb analysis.

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